DISTANCE LEARNING PROGRAMME

# NURTURE TEST SERIES / JOINT PACKAGE COURSE

## TARGET : JEE (MAIN)

Test Type : **ALL INDIA OPEN TEST** (MAJOR)    Test Pattern : JEE-Main

### ANSWER KEY

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Que.** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **Ans.** | 4 | 4 | 4 | 3 | 4 | 2 | 2 | 3 | 3 | 3 | 1 | 4 | 3 | 1 | 1 | 2 | 3 | 2 | 1 | 3 |
| **Que.** | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| **Ans.** | 4 | 1 | 3 | 2 | 3 | 4 | 2 | 3 | 3 | 4 | 4 | 2 | 4 | 3 | 1 | 1 | 4 | 4 | 2 | 1 |
| **Que.** | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| **Ans.** | 3 | 1 | 1 | 1 | 4 | 4 | 1 | 3 | 2 | 3 | 3 | 1 | 4 | 2 | 2 | 1 | 4 | 2 | 2 | 4 |
| **Que.** | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| **Ans.** | 1 | 3 | 2 | 4 | 2 | 4 | 3 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 4 | 2 |
| **Que.** | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | | | | | | | | | | |
| **Ans.** | 1 | 4 | 3 | 3 | 2 | 3 | 2 | 2 | 1 | 4 | | | | | | | | | | |

### HINT – SHEET

**1. Ans. (4)**

**Sol.** Velocity of ball at the lowest point = $\sqrt{2gL}$

for completing circle about peg

required minimum velocity = $\sqrt{5g(L-d)}$

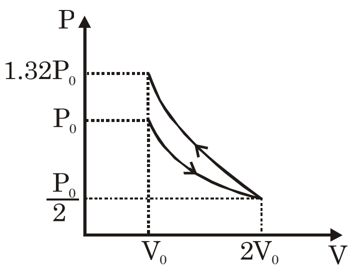
so $\sqrt{2gL} = \sqrt{5g(L-d)}$

$2L = 5L - 5d$

$d = \dfrac{3L}{5}$

**2. Ans. (4)**

**Sol.**




for adiabatic compression

$PV^{\gamma}$ = constant

$$\frac{P_0}{2}(2V_0)^{\gamma} = 1.32P_0(V_0)^{\gamma}$$

$2^{\gamma-1} = 1.32$

$\gamma = 1.4$ (diatomic)

for isothermal process

$\Delta T = 0$

so $\Delta KE = 0$

for adiabatic process

initial temperature = $\dfrac{P_0V_0}{nR}$

Final temperature = $\dfrac{1.32P_0V_0}{nR}$

Temperature increased by 1.32 times of initial so the KE.

**3. Ans. (4)**

**Sol.** $I_1 = \frac{mR^2}{2}$

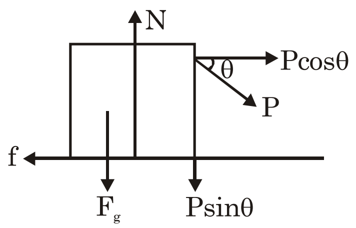
$I_2 = \frac{m}{2}R^2(1-\alpha\Delta T)^2$

$I_2 = I_1(1-\alpha\Delta T)^2$

$I_2 \approx I_1(1-2\alpha\Delta T)$

**4. Ans. (3)**

**Sol.**




when it is about to slip

$f = \mu_S N = P\cos\theta$

$N = F_g + P\sin\theta$

so $\mu_S(F_g + P\sin\theta) = P\cos\theta$

$\mu_S F_g = P\cos\theta - P\mu_S\sin\theta$

$P = \frac{\mu_S F_g}{\cos\theta - \mu_S\sin\theta}$

$P = \frac{\mu_S F_g \sec\theta}{1-\mu_S\tan\theta}$

**5. Ans. (4)**

**Sol.** $(2n+1)\frac{\lambda}{4} = L$

$\lambda = \frac{4L}{(2n+1)}$

$f = \frac{V}{\lambda} = (2n+1)\frac{V}{4L}$

$10 \le f \le 5000$

$f = (2n+1)\frac{340}{4\times1.2}$

$f = 70.83(2n+1)$

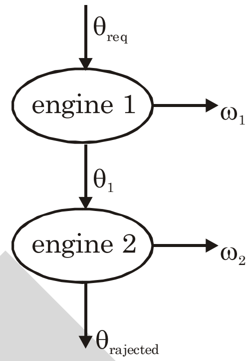
$5000 = 70.83(2n+1)$

$n \approx 35$

for the frequency range 10 – 5000 Hz possible frequencies are almost 35.

**6. Ans. (2)**

**Sol.** $e_1 = \frac{\omega_1}{\theta_{req}}$ ......(1)




$e_2 = \frac{\omega_2}{\theta_1}$ .....(2)

$e_{net} = \frac{\omega_1+\omega_2}{\theta_{req}}$

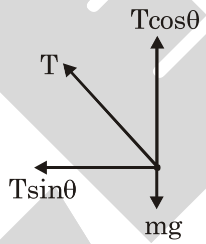
substituting $e_1$ and $e_2$ from equation (1) and (2)

$e_{net} = \frac{\theta_{req}e_1 + \theta_{req}e_2 - \omega_1 e_2}{\theta_{req}}$

$e_{net} = e_1 + e_2 - e_1 e_2$

**7. Ans. (2)**

**Sol.**




$\ell\sin\theta = R$

$T\cos\theta = mg$ ....(i)

$T\sin\theta = \frac{mv^2}{R}$

$T\sin\theta = \frac{mv^2}{\ell\sin\theta}$ ....(ii)

(ii)/(i)

$\frac{v^2}{\ell g\sin\theta} = \tan\theta$

$v = \sqrt{\ell g\sin\theta\tan\theta}$

Angular momentum

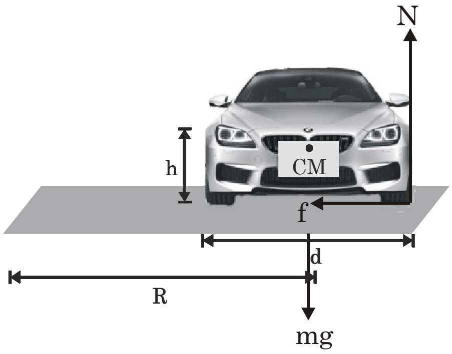
$L = mvR$

$L = m\sqrt{\ell g\sin\theta\tan\theta}\,\ell\sin\theta$

$L = \left(\frac{m^2\ell^3 g\sin^4\theta}{\cos\theta}\right)^{1/2}$

**8. Ans. (3)**

**Sol.**




$f = \frac{mv^2}{R}$ ...(i)

N = mg ....(ii)

Torque before amount COM

$f \times h = N\frac{d}{2}$

$f = N\frac{d}{2h}$

from (2)

so $f = \frac{mgd}{2h}$ ...(iii)

From (i) & (iii)

$\frac{mgd}{2h} = \frac{mv^2}{R}$

$V_{max} = \sqrt{\frac{Rgd}{2h}}$

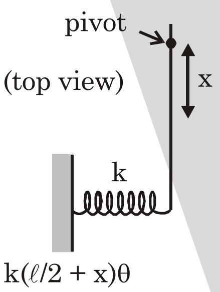
**9. Ans. (3)**

**Sol.** $L_A = mv\ell + I\omega$

$L_B = -mv\ell + I\omega$

**10. Ans. (3)**

**Sol.**




So Torque $= k\left(\frac{\ell}{2}+x\right)\theta\left(\frac{\ell}{2}+x\right)$

$\tau = -k\left(\frac{\ell}{2}+x\right)^2\theta$

$\left(\frac{m\ell^2}{12}+mx^2\right)\alpha = -k\left(\frac{\ell}{2}+x\right)^2\theta$

$\alpha = \frac{-k\left(\frac{\ell}{2}+x\right)^2}{m\left(\frac{\ell^2}{12}+x^2\right)}\theta$

$\omega^2 = \frac{k}{m}\left(\frac{\left(\frac{\ell}{2}+x\right)^2}{\frac{\ell^2}{12}+x^2}\right)$

$\omega = \sqrt{\frac{k}{m}}\sqrt{\frac{\left(\frac{\ell}{2}+x\right)^2}{\frac{\ell^2}{12}+x^2}}$

ω is minimum at

$x = \frac{\ell}{6}$ so the frequency.

**11. Ans. (1)**

**Sol.** Applying energy conservation

$mg\frac{L}{2} = \frac{1}{2}mv^2$

$v = \sqrt{gL}$

**12. Ans. (4)**

**Sol.** By W.E.T

Speed of projection $= 2\sqrt{gR\cos\theta}$

Range $= \frac{2\left(2\sqrt{gR\cos\theta}\right)^2\sin\theta\cos\theta}{g}$

$\frac{d}{d\theta}(\text{Range}) = 8R[-2\cos\theta\sin^2\theta + \cos^3\theta] = 0$

$\tan\theta = \frac{1}{\sqrt{2}}$

**13. Ans. (3)**

**Sol.** Force is developed due to property of liquid called surface tension.

**14. Ans. (1)**

**Sol.** By W.E.T

$mg(h-R) = \frac{1}{2}\left(\frac{7}{5}mv^2\right)$

$v^2 = \frac{20}{7}gR$ ....(i)

$N = \frac{mv^2}{R}$

$= \frac{20}{7}mg$

**15. Ans. (1)**

**Sol.** Since water falls at maximum distance from wall so the hole should be made at h.

For toppling, $\rho a v^2 \times h \geq \pi r^2 2h\rho g \times r$

$\Rightarrow \rho a 2gh \times h \geq \pi r^2 2h\rho g r \Rightarrow h \geq \frac{\pi r^3}{a}$

**16. Ans. (2)**

**Sol.** Final common temperature

$$T = \frac{m_1S_1T_1 + m_2S_2T_2}{m_1S_1 + m_2S_2}$$

because $m_1 = m_2$ given

$$\therefore \quad T = \frac{S_AT_A + S_wT_w}{S_A + S_w}$$

& $S_A = \frac{1}{2}S_w$ given,

$$T = \frac{\frac{T_A}{2} + T_w}{\frac{1}{2}+1} = \left(\frac{T_A + 2T_w}{3}\right)$$

$T = \left(\frac{T_A}{3} + \frac{2T_w}{3}\right)$ & This T is nearer to $T_w$

Method-II

$S_A = \frac{1}{2}S_w$ given,

so, water has more thermal inertia, hence in mixture, it will loose less temperature & final temperature will be nearer to $T_w$.

**17. Ans. (3)**

**Sol.** $f' = \left(\frac{V \pm V_0}{V \pm V_S}\right)f$

$f_1 = \left(\frac{V \pm V_0}{V}\right)f$

$f_2 = \left(\frac{V}{V \pm V_S}\right)S$

$\lambda_2 = \left(\frac{V \pm V_S}{f}\right)$

$V_0$ = velocity of listener; $V_S$ = Velocity of source.

But if $\vec{V}_S = \vec{V}_0$ then f' = f

Thus statement-3 is not always correct

**18. Ans. (2)**

**Sol.** $I = \frac{P}{4\pi r^2}$

$dB = 10 \log \frac{I}{I_0}$

**19. Ans. (1)**

**Sol.** Frequency depends on external source ; therefore it doesn't change.

**20. Ans. (3)**

**Sol.** $y_1 = A \sin (\omega t - kx)$

$y_2 = A \sin (\omega t - kx + 90°)$

$y_1 + y_2 = \sqrt{2} A \sin (\omega t - kx + 45°)$

**21. Ans. (4)**

**Sol.** $P = \frac{1}{2}\mu A^2\omega^2 V$

Also $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho \times Area}}$

$= \sqrt{\frac{T}{\rho \times \frac{\pi}{4}d^2}}$

$\Rightarrow$ When $T \to 4$ times

$V \to 2$ times

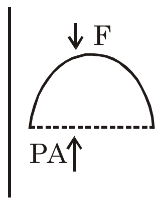
$\Rightarrow P \to 2$ times $\Rightarrow$ (C) correct

Also

$$P = \frac{1}{2}\left(\rho \times \frac{\pi}{4}d^2\right)A^2\omega^2\sqrt{\frac{T}{\rho \times \frac{\pi}{4}d^2}} \propto d$$

(4) correct

**22. Ans. (1)**

**Sol.** 


$PA - F = F_B = \frac{2\pi}{3}r^3\rho_1 g$

$(P_0 + \rho_1 gh)\,\pi r^2 - F$

$= \frac{2\pi}{3}r^3\rho_1 g$

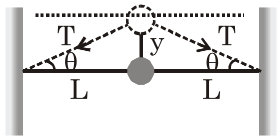
$F = P_0\pi r^2 + \left(h - \frac{2}{3}r\right)\pi r^2\rho_1 g$

**23. Ans. (3)**

**Sol.** Use energy conservation.

**24. Ans. (2)**

**Sol.**




$2T\sin\theta = ma$

$\sin\theta \approx \tan\theta$

$$2T\left(-\frac{y}{L}\right) = ma$$

$$a = -\frac{2T}{mL}y$$

$$\omega = \sqrt{\frac{2T}{mL}}$$

**25. Ans. (3)**

**Sol.** By W.E.T

$mgR(1 - \cos\theta)$

$$= \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\omega^2 \quad ...(i)$$

$$v = \omega R \quad ...(ii)$$

**26. Ans. (4)**

**Sol.** $\mu = \left(\frac{\mu_L - \mu_o}{L}\right)x + \mu_0$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\frac{dx}{dt} = \frac{\sqrt{T}}{\left[\left(\frac{\mu_L - \mu_0}{L}\right)x + \mu_0\right]^{1/2}}$$

$$\int_0^L \left[\left(\frac{\mu_L - \mu_0}{L}\right)x + \mu_0\right]^{1/2} dx = \int_0^t \sqrt{T}dt$$

$$\frac{2L}{3}\frac{\left(\sqrt{\mu_L}\right)^3 - \left(\sqrt{\mu_0}\right)^3}{\mu_L - \mu_0} = \sqrt{T}t$$

**27. Ans. (2)**

**Sol.** Apply conservation of angular momentum

$L_{final} = L_{initial}$

**28. Ans. (3)**

**Sol.** Stress = $\frac{mg}{A}$

$$\frac{(\text{stress})_u}{(\text{stress})_\ell} = \frac{(m_u + m_\ell)g/A_u}{(m_\ell)g/A_\ell}$$

$$= \frac{(d_uA_uL_u + d_\ell A_\ell L_\ell)/A_u}{(d_\ell A_\ell L_\ell)/A_\ell}$$

$$= \left(\frac{d_uA_uL_u}{d_\ell A_\ell L_\ell} + 1\right)\frac{A_\ell}{A_u} = \frac{5}{6}$$

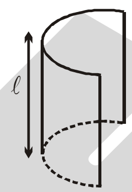
**29. Ans. (3)**

**Sol.** As the mass of solid sphere will be more than hollow sphere $\frac{dT}{dt} \propto \frac{1}{m}$ hence $T_1 > T_2$

**30. Ans. (4)**

**Sol.** $T = p\cdot(2R\ell)$

$$\sigma = \frac{T}{2t\ell}$$




$$= \frac{pR}{t} \Rightarrow p = \frac{\sigma t}{R}$$

**61. Ans. (1)**

m and n are the roots of

$a(\ell + x)^2 + 2b\ell x + c = 0$

$\Rightarrow \quad ax^2 + (2a + 2b)\ell x + (a\ell^2 + c) = 0$

$$\Rightarrow \quad m.n = \frac{a\ell^2 + c}{a} = \ell^2 + \frac{c}{a}$$

**62. Ans. (3)**

$$b = \frac{2ac}{a + c}$$

$$D = 4(b^2 - ac) = -4ac\left(\frac{a - c}{a + c}\right)^2 < 0$$

**63. Ans. (2)**

If $x = 2n + y,\ n \in I,\ y \in [0, 1)$

Then $[x] = 2n, \left[\frac{x}{2}\right] = n, \left[\frac{x+1}{2}\right] = n$

$$\Rightarrow \quad \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] = [x]$$

If $x = (2n + 1) + y,\ y \in [0, 1),\ n \in I$

$$[x] = 2n + 1,\ \left[\frac{x}{2}\right] = n, \left[\frac{x+1}{2}\right] = n + 1$$

$$\Rightarrow \quad \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] = [x]\ \forall x \in R^+$$

**64. Ans. (4)**

$$f(x)+f\left(\frac{1}{x}\right)=1$$

$$\Rightarrow \quad x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$$

$$x+\frac{1}{x}=[x]+\left[\frac{1}{x}\right]+1=\text{integer} \quad \ldots\ldots(1)$$

$$\Rightarrow \quad x+\frac{1}{x}=n$$

$$x^2-nx+1=0 \Rightarrow x=\frac{n\pm\sqrt{n^2-4}}{2}$$

But $n \neq 2, -2$ as it does not satisfy (i)

$\Rightarrow$ n can be any integer in $(-\infty, -2) \cup (2, \infty)$

So infinite solutions.

**65. Ans. (2)**

$t_r = r(n - (r - 1)) = nr - r^2 + r$

$$\sum t_r = (n+1)\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2 = \frac{n(n+1)(n+2)}{6}$$

**66. Ans. (4)**

Required number of ways is

Number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 = 12$.

$\Rightarrow \quad {}^{12+4-1}C_{4-1} = {}^{15}C_3 = 455.$

**67. Ans. (3)**

Let $x_1 = y_i + i, i = 1, 2, \ldots\ldots, r$.

$y_1 + y_2 + \ldots\ldots + y_r = n - (1 + 2 + \ldots\ldots + r)$

$$= n - \frac{r(r+1)}{2} = \lambda, \ y_i \geq 0$$

Number of integral solutions

$$= {}^{\lambda+r-1}C_{r-1} = {}^{\lambda+r-1}C_{\lambda}$$

**68. Ans. (1)**

Let $a - 3 = b$ so that $a - 2 = 1 + b$.

The given equation is

$$\sum_{i=0}^{2n} \lambda_i (1+b)^i = \sum_{i=0}^{2n} \mu_i b^i$$

$$\Rightarrow \quad \lambda_0 + \lambda_1(1+b) + \ldots + (1+b)^n + \ldots + (1+b)^{2n}$$

$$= \mu_0 + \mu_1 b + \ldots + \mu_n b^n + \ldots + \mu_{2n} b^{2n},$$

$(\because \ \lambda_1 = 1 \ \forall \ i \geq n)$

Comparing coefficient of $b^n$ we get

${}^{n}C_n + {}^{n+1}C_n + \ldots.+ {}^{2n}C_n = \mu_n = {}^{2n+1}C_n$.

**69. Ans. (2)**

Degree of LHS = 3n + 5

Degree of RHS = 2

$\Rightarrow \quad 3n + 5 = 2 \quad \Rightarrow \quad n = -1$

**70. Ans. (2)**

$$\left|\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\right| = \left|\frac{1-\sin\theta+1+\sin\theta}{|\cos\theta|}\right|$$

$$=\frac{2}{-\cos\theta} = -2\sec\theta$$

**71. Ans. (2)**

$$f(x)=\frac{1}{2}(1-\cos 2x)+\frac{1}{2}\left(1-\cos\left(2x+\frac{2\pi}{3}\right)\right)$$

$$+\frac{1}{2}\left(\cos\left(2x+\frac{\pi}{3}\right)+\cos\frac{\pi}{3}\right)$$

$$=\frac{5}{4}-\frac{1}{2}\left\{2\cos\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}-\cos\left(2x+\frac{\pi}{3}\right)\right\}=\frac{5}{4}$$

$$\Rightarrow \quad g\left(f\left(\frac{\pi}{8}\right)\right)=g\left(\frac{5}{4}\right)=1$$

**72. Ans. (2)**

$\sqrt{\sin x} = -\cos x \qquad (\cos x \leq 0)$

$\Rightarrow \quad \sin x = \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \quad \sin^2 x + \sin x - 1 = 0$

$$\sin x = \frac{\sqrt{5}-1}{2}, \ k \in \text{IInd quadrant}$$

$$x = \pi - \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$$

**73. Ans. (3)**

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$=\frac{2\sin\left(\frac{\pi}{2}\right)}{\cos\frac{\pi}{2}+\cos\theta}=\frac{2}{\cos\theta}=\frac{2}{\left(\frac{3}{5}\right)}=\frac{10}{3}$$

**74. Ans. (3)**

$$\cos\left(\frac{\theta}{2}\right)=\frac{x}{3}.\frac{y}{2}-\sqrt{\left(1-\frac{x^2}{9}\right)\left(1-\frac{y^2}{4}\right)}$$

$$4x^2+9y^2+36\cos^2\frac{\theta}{2}-12xy\cos\frac{\theta}{2}=36$$

$$4x^2-12xy\cos\frac{\theta}{2}+9y^2=36\sin^2\frac{\theta}{2}=18(1-\cos\theta)$$

**75. Ans. (1)**

$$a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}+C$$

$$=a\left(\frac{(s-c)(s-a)}{ac}\right)+\frac{b(s-b)(s-c)}{bc}+C$$

$$=\left(\frac{s-c}{c}\right)(s-a+s-b)+c$$

$$=\left(\frac{s-c}{c}\right)(c)+c=s$$

**76. Ans. (1)**

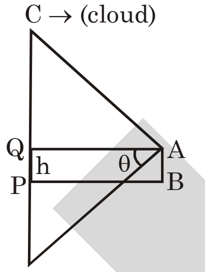
PB $\rightarrow$ surface of lake

PC = PD = H

QA = (H – h)cotθ

= (H + h)cotθ

D $\rightarrow$ (Reflection)




$$H=h\left(\frac{\cot\theta+\cot\phi}{\cot\theta-\cot\phi}\right)$$

$$=\frac{h\sin(\theta+\phi)}{\sin(\phi-\theta)}$$

**77. Ans. (2)**

$12x - 5y + 7 = 0$

$4x - 3y + 1 = 0$

$a_1a_2 + b_1b_2 > 0$

$\Rightarrow$ obtuse angle bisector.

$$\frac{12x-5y+7}{13}=\frac{4x-3y+1}{5}$$

$\Rightarrow$ $4x + 7y + 11 = 0$

**78. Ans. (2)**

$f(2x+3y,\ 2x-7y)=20x$

$2x + 3y = u$

$2x - 7y = v$

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$10y = u - v$

$$y=\frac{u-v}{10}$$

$$2x = u - 3y = u-\frac{(3u-3v)}{10}=\frac{7u+3v}{10}$$

$$f(u,\ v)=\frac{20(7u+3v)}{20}=7u+3v$$

$f(x, y) = 7x + 3y$

**79. Ans. (4)**

Let A(a, b) $\Rightarrow$ 9a + 7b = 28

Let centroid is (x, y)

then 3x = a + 4 + 6

3y = b + 7 + 1

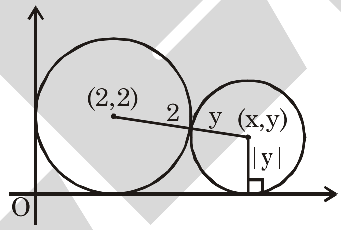
$\Rightarrow$ a = 3x – 10

b = 3y – 8

$\Rightarrow$ 9(3x – 10) + 7(3y – 8) = 28

$\Rightarrow$ 9x + 7y – 58 = 0 which is parallel to 9x + 7y = 28

**80. Ans. (2)**




$(2 - x)^2 + (2 - y)^2 = (y + 2)^2$

$x^2 - 4x - 8y + 4 = 0$

**81. Ans. (1)**

$$\tan^{-1}\left(\frac{x+1+\frac{1}{x-1}}{1-\frac{x+1}{x-1}}\right)=2\tan^{-1}\frac{4}{3}$$

$$\Rightarrow\quad x=4\sqrt{\frac{3}{7}}$$

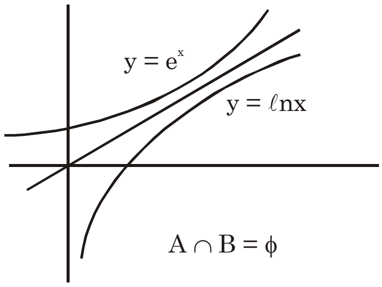
**82. Ans. (4)**

$$=\frac{1}{a}\begin{vmatrix}-a & a\cos C & a\cos B\\ \cos C & -1 & \cos A\\ \cos B & \cos A & -1\end{vmatrix}$$

$$= R_1 \rightarrow R_1 + bR_2 + cR_3$$

$$=\frac{1}{a}\begin{vmatrix}0 & 0 & 0\\ \cos C & -1 & \cos A\\ \cos B & \cos A & -1\end{vmatrix}=0$$

**83. Ans. (3)**



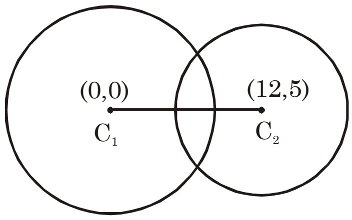

**84. Ans. (3)**

$|a| \le b$ and $|b| \le c$

$\Rightarrow \quad |a| \le c$

$\Rightarrow$ relation is transitive

**85. Ans. (2)**




$r_1 = 2,\ r_2 = \sqrt{169 - k^2}$

$C_1C_2 = 13$

(1) $169 - k^2 \ge 0 \Rightarrow \quad k \in [-13, 13]$

(2) $13 < 2 + \sqrt{169 - k^2}$

$\Rightarrow \quad 121 < 169 - k^2$

$k^2 < 48 \quad \Rightarrow \quad k \in \left(-\sqrt{48}, \sqrt{48}\right)$

(3) $\left|\sqrt{169 - k^2} - 2\right| < 13 \Rightarrow k^2 < 48$

$\Rightarrow \quad k = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6$

**86. Ans. (3)**

$y = (101)^{50} = (100 + 1)^{50}$

$(99)^{50} = (100 - 1)^{50}$

$(101)^{50} - (99)^{50}$

$= 2\left({}^{50}C_0 100^{50} + \ldots\ldots + {}^{50}C_{50}\right) > 100^{50}$

$(101)^{50} > 100^{50} + 99^{50}$

$y > x$

**87. Ans. (2)**

$\sum_{r=0}^{10} {}^{20}C_r$

$= (C_0 + C_1 + \ldots + C_9) + C_{10} + (C_{11} + \ldots + C_{20}) = 20^{20}$

$2(C_0 + \ldots\ldots + C_{10}) = 2^{20} + {}^{20}C_{10}$

$C_0 + \ldots\ldots + C_{10} = 2^{19} + \frac{{}^{20}C_{10}}{2}$

**88. Ans. (2)**

Required number of ways

$= {}^5C_2 4!4! = 5760$

**89. Ans. (1)**

Let $\log_5 x = t$

$t^2 + t - 2 < 0$

$t \in (-2, 1)$

$x \in \left(\frac{1}{25}, 5\right)$

**90. Ans. (4)**

$x^2 + (a + b)x + ab < 0$

$(x + a)(x + b) < 0$

$x \in (-b, -a)$