ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

DISTANCE LEARNING PROGRAMME

JEE (Main)
MOCK TEST

JEE(Main) : LEADER TEST SERIES

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	D	A	B	D	D	B	B	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	D	B	C	C	B	C	C	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	9	6	2	120	2					

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	C	C	C	B	D	D	A	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	B	A	B	A	C	B	C	B	D
SECTION-II	Q.	1	2	3	4	5					
	A.	75	62	2	5	6					

PART-3 : MATHEMATICS

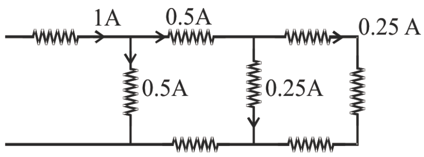
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	C	A	D	A	B	D	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	B	A	B	D	C	B	B	B	D
SECTION-II	Q.	1	2	3	4	5					
	A.	2549	5	36	22	2					

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Ans (D)



$$i = \frac{V}{R_{eq}} = \frac{9}{9} = 1\text{ A}$$

2. Ans (D)

For diamagnetic

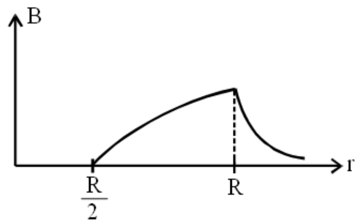
$\chi \propto T^0$

3. Ans (D)

$r < \frac{R}{2}$, $B = 0$

$r > R$, $B = \frac{\mu_0 I}{2\pi R}$

$$\frac{R}{2} < r < R,\ B = \left[\frac{r^2 - \left(\frac{R}{2}\right)^2}{R^2 - \left(\frac{R}{2}\right)^2}\right]$$



4. Ans (A)

Given $e = E_0 \sin \omega t$

$i = I_0 \sin(\omega t - \phi)$

So phase difference = ϕ

power factor = $\cos\phi$

By formula

Average power

$(P_{av}) = V_{rms} I_{rms} \cos\phi$

$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$

$= \frac{E_0 I_0}{2} \cos\phi \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$

$= \frac{E_0 I_0}{2} \cos\phi$

5. Ans (B)

$\because P \longrightarrow$ same

$\varepsilon_1 I_1 = \varepsilon_2 I_2$

$\frac{I_1}{I_2} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{L_2}{L_1} = \frac{1}{4}$

$\left\{\because \frac{dI}{dt} \rightarrow \text{same}\right\}$

6. Ans (D)

The equation of electric field occurring in Y-direction $E_y = 66 \cos 2\pi \times 10^{11\left(t - \frac{x}{c}\right)}$

Therefore, for the magnetic field in Z-direction

$B_z = \frac{E_y}{c} = \left(\frac{66}{3 \times 10^8}\right) \cos 2\pi \times 10^{11} \left(t - \frac{x}{c}\right)$

$= 22 \times 10^{-8} \cos 2\pi \times 10^{11\left(t - \frac{x}{c}\right)}$

$= 2.2 \times 10^{-7} \cos 2\pi \times 10^{11} \left(t - \frac{x}{c}\right)$

7. Ans (D)

NCERT XI, Pg. 266

The distance of n^{th} bright fringe from central bright fringe $x_n = \frac{n\lambda D}{d}$

$X_3 = \frac{3 \times 700 \times 10^{-9} \times 2}{1 \times 10^{-3}}$

$x_3 = 4.2$ mm

Distance between on either side = $2x_3$

$= 2 \times 4.2 = 8.4$ mm

8. Ans (B)

NCERT-XI, Pg.No. 234

9. Ans (B)

$f = \frac{v}{2\ell} = 400$ Hz

$f' = \frac{v}{4\left(\frac{\ell}{2}\right)} = 400$ Hz

10. Ans (A)

$dQ = dU + dW$

$5 = Q + W_{AB} + W_{BC} + W_{CA}$

$5 = (2 - 1) + 0 + W_{CA}$

$W_{CA} = -5$ Joule

11. Ans (C)

PT^2 = constant

$\frac{PV}{T}$ = constant

$\therefore \left(\frac{T}{V} \text{constant}\right) T^2$ = constant

constant $V = T^3$

constant $\frac{dV}{dT} = 3T^2$

$\frac{dV}{dT} = \frac{3T^2 dT}{T}.\rho$

$\frac{1}{V} \cdot \frac{dV}{dT} = \frac{3}{T} \cdot$constant

12. Ans (D)

$e \longrightarrow V_0$

$E = -E_0\hat{i}$

$\lambda_0 = \frac{h}{mv_0} \Rightarrow v = v_0 + \frac{eE_0 t}{m}$

$\lambda = \frac{h}{mv} = \frac{h}{m\left(v_0 + \frac{eE_0}{m}t\right)}$

$\Rightarrow \lambda' = \frac{h}{mv_0\left(1 + \frac{eE_0}{mv_0}t\right)}$

$\Rightarrow \lambda' = \frac{\lambda_0}{\left(1 + \frac{eE_0}{mv_0}t\right)}$

13. Ans (B)

$I_L = \frac{5}{1000} = 5$mA

14. Ans (C)

Energy released = $(\Delta m)_{amu} \times 931.5$ MeV

$= (m_u - m_{Th} - m_{He})_{amu} \times 931.5$ MeV

$= 0.0044 \times 931.5$ MeV $= 4.0986$ MeV

15. Ans (C)

$$a = \frac{-kx}{2} = \frac{-12x}{2} = -6x$$

$$\frac{vdv}{dx} = -6x$$

$$\int_4^v vdv = -\int_{\frac{1}{2}}^{3/2} 6xdx$$

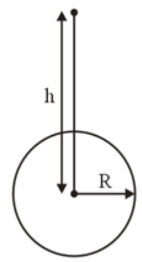
$$\frac{v^2 - 4^2}{2} = -\frac{6}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$v^2 - 16 = -6\left(\frac{9}{4} - \frac{1}{4}\right)$$

$$v^2 = 16 - 6 \times 2 = 4$$

$$V = 2 \text{ m/s}$$

16. Ans (B)



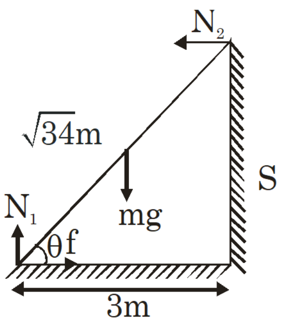
$$-\frac{GMm}{R} + \frac{1}{2}m\lambda^2 V_e^2 = -\frac{GMm}{h}$$

$$-\frac{GMm}{R} + \frac{1}{2}\lambda^2 \frac{2GMm}{R} = -\frac{GMm}{h}$$

$$\frac{\lambda^2}{R} - \frac{1}{R} = \frac{-1}{h} \Rightarrow \frac{1}{h} = \frac{1-\lambda^2}{R}$$

$$\Rightarrow h = \frac{R}{1-\lambda^2}$$

17. Ans (C)



$$f = N_2$$

$$N_1 = mg$$

$$N_2 \times \ell \sin\theta = mg\frac{\ell}{2}\cos\theta$$

$$N_2 = \frac{mg}{2}\cot\theta$$

18. Ans (C)

L_0 = angular momentum of shell about O.

As shell is rolling

so $V_{cm} = \omega R$

$$L_0 = mV_{cm} R + I\omega$$

$$= 1 \times \omega R \times R + \frac{2}{3}R^2\omega$$

$$= \frac{5}{3}R^2\omega$$

so a = 5

19. Ans (D)

$$y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)}$$

$$\Rightarrow 10 = 20 - \frac{(10)(100)}{u^2}$$

$$u = 20$$

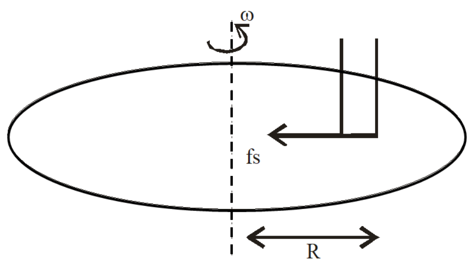
$$T = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$$

$$\vec{v} = 10\sqrt{2}\hat{i} + (10\sqrt{2} - 10(2)]\hat{j}$$

$$\text{Momentum } \vec{p} = M\vec{v} = 100\sqrt{2}\hat{i} + (100\sqrt{2} - 200)\hat{j}$$

20. Ans (B)

For beaker to move with disc



$$f_s = m\omega^2 R$$

We know that $f_S \leqslant f_{s\ max}$

$$m\omega^2 R \leqslant \mu mg$$

$$R \leqslant \frac{\mu g}{\omega^2}$$

PART-1 : PHYSICS

SECTION-II

1. **Ans (9)**

Fill the empty space with $+\rho$ and $-\rho$ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\,\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\,\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

2. **Ans (6)**

$6\ \mu C$, Hint : the p.d. across R_2 is 4 volt. The total charge in the two parallel arms is $12\ \mu C$.
Thus in each row, $q = 6\ \mu C$.

3. **Ans (2)**

$$\gamma = \frac{\Delta V}{V\,\Delta T}$$

Given $\frac{\Delta V}{V} = 0.12\%$ or 0.0012

$$\gamma = \frac{0.0012}{20} = 6 \times 10^{-5}/^\circ C$$

Since, $\alpha = \frac{\gamma}{3} = 2 \times 10^{-5}/^\circ C$

4. **Ans (120)**

By energy conservation

PE = KE

$$mg\left(H + \frac{H}{2}\right) = \frac{1}{2}kx^2 \left(x = \frac{H}{2}\right)$$

$$0.100 \times 10 \times \frac{3}{2}(0.10) = \frac{1}{2}k(0.05 \times 0.05)$$

$$k = \frac{3 \times 0.10}{0.05 \times 0.05}$$

$$= \frac{3 \times 1000}{25} = 120\text{N/m}$$

5. **Ans (2)**

$$V = \frac{GM}{r} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$$

PART-2 : CHEMISTRY

SECTION-I

1. **Ans (C)**

Moles at eqm $\underset{2-x}{A_2(g)} + \underset{4-x}{B_2(g)} \rightleftharpoons \underset{2x}{2AB(g)}$

$$K_c = \frac{4x^2}{(2-x)(4-x)}$$

$$\Rightarrow x = \frac{32}{24} = 1.33 \text{ mole}$$

$$[AB(g)] = \frac{2 \times 1.33}{4} = 0.66\,M$$

2. **Ans (B)**

Given $t_{67\%} - t_{33\%} = 30$

$$\left(\frac{2.303}{k}\log\frac{a}{a/3}\right) - \left(\frac{2.303}{k}\log\frac{a}{2a/3}\right) = 30$$

$$\frac{2.303}{k}[\log 3 - \log 3 + \log 2] = 30$$

$$k = \frac{2.303 \times 0.3}{30} = \frac{2.303}{100}$$

$$t_{25\%} = \frac{2.303}{2.303} \times 100 \log\frac{a}{3a/4}$$

$$t_{25\%} = 100 \times \log\frac{4}{3} = 100 \times 0.123 = 12.3 \text{ min.}$$

3. **Ans (C)**

$$pH = \frac{1}{2}\left(p^{kw} + p^{ka} + \log c\right)$$

$$c = \frac{0.1 \times 25}{50} = 0.05$$

$$pH = \frac{1}{2}(14 + 3.74 + \log 0.05)$$

$$= \frac{1}{2}(17.74 - 2 + 0.7)$$

$$= 8.22$$

4. Ans (C)

0.05 mol XY_2 = 5 gm

1 mol $XY_2 = \frac{5}{0.05} = 100$

$X + 2Y = 100$ ...(i)

3.01×10^{23} molecule X_2Y_3 = 85 gm

1 mol $X_2Y_3 = N_A$ molecule = 170 gm

$2X + 3Y = 170$ ...(ii)

On solving,

X = 40, Y = 30

5. Ans (C)

$C_6H_{12}O_6 \longrightarrow 2C_3H_4O_3 + 4H^+ + 4e^-$

n-factor = 4

6. Ans (B)

(b) For

$H_2(g) + O_2(g) \longrightarrow H_2O_2(l)$

$\Delta_f H°(H_2O_2, l) = \Delta_r H_3^o + \frac{\Delta_r H_2^o}{2} - \frac{\Delta_r H_1^o}{2}$

7. Ans (D)

$(\Delta G)_{T,P} < 0$ for a spontaneous process

8. Ans (D)

$K_2\overset{+6}{Mn}o_4 \xrightarrow[\text{(oxidation)}]{} K\overset{+7}{Mn}o_4$

(1) Passing O_3 gas (strong oxidising agent)

$2K_2Mno_4 + O_3 + H_2O \longrightarrow 2KMno_4 + 2KoH + O_2$

(2) Passing Cl_2 gas (Oxidising Agent)

$2K_2Mno_4 + Cl_2 \longrightarrow 2KMno_4 + 2KCl$

(B) Electrolytic oxidation

$Mno_4^{2-} \rightarrow Mno_4^- + e^{\Theta}$

Hence, All the methods can be used to oxidise

K_2Mno_4 to $KMno_4$.

9. Ans (A)

$Yb^{2+} \longrightarrow [Xe]\ 4f^{14} \longrightarrow$ diamagnetic

$Ce^{2+} \longrightarrow [Xe]\ 4f^1\ 5d1 \longrightarrow$ paramagnetic

$Sm^{2+} \longrightarrow [Xe]\ 4f^6 \longrightarrow$ paramagnetic

$Eu^{2+} \longrightarrow [Xe]\ 4f^7 \longrightarrow$paramagnetic

10. Ans (C)

1st electron affinity of non−metals is generally negative.

11. Ans (C)

IE of Ga > IE of Al

12. Ans (B)

If hunds rule does not hold good all species with even number of e^- are diamagnetic.

13. Ans (A)

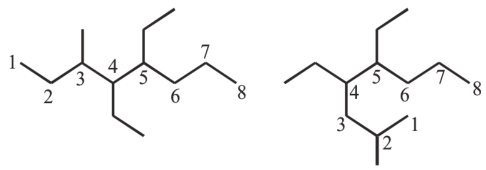
Concept en ≡ bidentate ligand

Co (+3) ≡ 6 (C.N.)

$[CoCl_2en_2]\ Cl$

A and B should be G.I.

14. Ans (B)



Locant position (3, 4, 5) & (2, 4, 5) differ along same parent chain.

15. Ans (A)

$R-\overset{\overset{O}{||}}{C}-OH \longrightarrow R-\overset{\overset{O}{||}}{C}-O^{\ominus}$

Equal contributor more stable

$Ph-OH \longrightarrow Ph-O^{\ominus}$

Unequal contributor
less stable

Acidic strength μ Stability of anion

Thus R — COOH > Ph − OH (Acid Strength)

When functional group (Source group) is same

Acid strength ∝ EWG ∝ $\frac{1}{\text{ERG}}$

Hence III > IV > II > I

16. **Ans (C)**

1. Asking About: Correct statement.

2. Concept: Carbylamine Test.

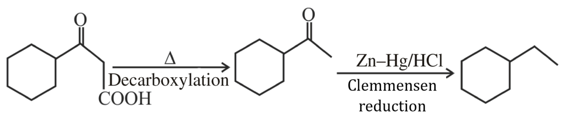
3. Solution/Explanation

Carbylamine Test is Test only for 1°/primary amine in which 1° Amine is treated with chloroform ($CHCl_3$) and KOH (Basic medium) then foul smelling compound R-NC (Isocyanide) is formed.

$R\text{-}NH_2 + CHCl_3 + KOH \longrightarrow R\text{-}NC$

Hence option **3** is correct answer.

17. **Ans (B)**



18. **Ans (C)**

$CH_3\text{–}C\equiv C\text{–}CH_3 \xrightarrow[\text{Pd-BaSO}_4]{\text{Pd-CaCO}_3} CH_3\text{–}\overset{H}{C}=\overset{H}{C}\text{–}CH_3$ (cis-2-butene)

$\downarrow Br_2$

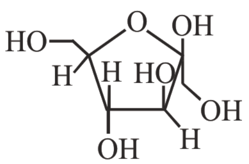
$CH_3\text{–}\underset{}{\overset{Br}{CH}}\text{–}\underset{Br}{CH}\text{–}CH_3$

($\pm$)Anti addition

19. **Ans (B)**

$Ph\text{—}\overset{O}{\overset{\|}{\underset{14}{C}}}\text{—}OH \xrightarrow{NaHCO_3} \underset{(A)}{CO_2} \xrightarrow[2H^+]{PhMgBr} Ph\text{—}COOH$

20. **Ans (D)**



β-keto furanose = β-keto fructose

PART-2 : CHEMISTRY

SECTION-II

1. **Ans (75)**

Initial moles of $Cr^{+3} = \frac{0.20 \times 250}{1000} = 0.05$

Remaining moles $= \frac{0.1 \times 250}{1000} = 0.025$

Used moles = 0.05 – 0.025 = 0.025 moles

$0.025 \times 3 = \frac{E}{96500} \times 96.5 \times t$

2. **Ans (62)**

$i = 1 - \left(1 - \frac{1}{4}\right)\alpha$

$i = 1 - \left(\frac{3}{4}\right) 0.8$

$i = 1 - 0.6 = 0.4$

$\because \Delta T_f = i \times K_f \times m$

$\therefore 0.6 = 0.4 \times 1.86 \times \frac{5/M\omega}{0.1}$

$\Rightarrow M_\omega = \frac{0.4}{0.6} \times \frac{1.86 \times 5}{0.1}$

$= \frac{4}{6} \times 18.6 \times 5$

$\boxed{M_\omega = 62}$

3. **Ans (2)**

p - p sideways over lapping is present in N ≡ N, O = O,

4. **Ans (5)**

$Cr_2O_7^{2-}$; $Na_2[Fe(CN)_5NO]$; $[Ni(CN)_4]^{2-}$; $[Sc(H_2O)_6]^{3+}$; $[Zn(NH_3)_4]^{2+}$

5. **Ans (6)**

a, c, d, e, h

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (A)**

$n(x) = 10,\ n(P(X)) = 2^{10}$

$A, B \in P(X) \Rightarrow n(s) = 2^{10} \times 2^{10} = 2^{20}$

Now, for n(A) = n(B)

$n(E) = \left({}^{10}C_0 \times {}^{10}C_0 + {}^{10}C_1 . {}^{10}C_1 + {}^{10}C_2 . {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10} . {}^{10}C_{10}\right)$

$= C_0^{\,2} + C_1^{\,2} + \ldots\ldots + C_{10}^{\,2} = {}^{20}C_{10}$

Read Probability $= \dfrac{n(E)}{n(s)} = \dfrac{{}^{20}C_{10}}{2^{20}}$

2. **Ans (B)**

The Correct Answer is (B)

3. **Ans (C)**

$A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow\ 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow\ 128\begin{bmatrix} x - y \\ -x + y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow\ x - y = \dfrac{1}{16} \quad \ldots(1)$

$\Rightarrow\ \&\ -x + y = \dfrac{1}{2} \quad \ldots(2)$

$\Rightarrow$ From (1) & (2) : No solution.

4. **Ans (A)**

$I_m\left(\dfrac{iz - 2}{z - i}\right) = -1$

$z = x + iy$

$I_m\left(\dfrac{ix - y - 2}{x + iy - i}\right) = -1$

$I_m\left(\dfrac{(y + 2) - ix}{x + i(y - 1)}\right) = 1$

$\left[\dfrac{[(y+2) - ix]\,[x - i(y - 1)]}{x^2 + (y - 1)^2}\right] = 1$

$-x^2 - (y - 1)(y + 2) = x^2 + (y - 1)^2$

$-x^2 - (y^2 + y - 2) = x^2 + (y^2 - 2y + 1)$

$2x^2 + 2y^2 - y = 1$

$x^2 + y^2 - \dfrac{1}{2}y = \dfrac{1}{2}$

$x^2 + \left(y - \dfrac{1}{4}\right)^2 = \dfrac{1}{2} + \dfrac{1}{16} = \dfrac{9}{16} = \left(\dfrac{3}{4}\right)^2$

5. **Ans (D)**

$\dfrac{(\alpha^{15} + \beta^{15}) + \alpha^2\beta^2(\alpha^{11} + \beta^{11})}{\alpha^{13} + \beta^{13}}$

$\dfrac{(\alpha^{13} + \beta^{13})(\alpha^2 + \beta^2)}{(\alpha^{13} + \beta^{13})}$

$= \alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= 27$

6. **Ans (A)**

(a) 5 ball are identical & 3 boxes are different so no. of ways each box can be filled such that no box remain empty.

$= {}^{5-1}C_{3-1} = {}^4C_2 = 6$

option s is correct match.

(b) Here balls are different & box are identical.

Case-1. 1 . 1 . 3 $\Rightarrow \dfrac{{}^5C_1 . {}^4C_1}{2!} = 10$

Case-2. 1 . 2 . 2 $\Rightarrow \dfrac{{}^5C_1 . {}^4C_2}{2!} = 15$

So total no. of ways = 25

Option q is correct match.

(c) Balls as wall as boxes are identical

Case-1 1 . 1 . 3

Case-2 1 . 2 . 2

So there are two cases only

Option p is correct match.

(d)

$3^5 - {}^3C_1 2^5 + {}^3C_2 1$

$243 - 96 + 3$

$= 150$

7. Ans (B)

(sinx cosy) (f(2x + 2y) – f(2x – 2y)) = (cosx siny)

(f(2x + 2y) + f(2x – 2y))

f(2x + 2y)(sin(x–y)) = f(2x – 2y)sin(x + y)

$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$

Put 2x + 2y = m, 2x – 2y = n

$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = K$$

$$\Rightarrow f(m) = K\sin\left(\frac{m}{2}\right)$$

$$\therefore f(x) = K\sin\left(\frac{x}{2}\right)$$

$$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$$

Put x = 0 ; $\frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$

$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$

$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$

$$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$$

$$= \frac{-24}{8} = -3$$

8. Ans (D)

$f'(x) = 6x^2 - 18ax + 12a^2$

$f'(x) = 6(x^2 - 3ax + 2a^2)$

roots are a, 2a

$p^2 = q \Rightarrow a^2 = 2a$

a = 2

$f(x) = 2x^3 - 18x^2 + 48x + 1$

f(3) = 37

9. Ans (A)

$$\lim_{x\to10}\frac{x^2(\alpha x) + (\gamma-1)\left(1+\frac{x^2}{1}\right)}{2x - \frac{8x^3}{6} - \beta x} = 3$$

$$\lim_{x\to0}\frac{(\gamma-1) + (\gamma-1)x^2 + \alpha x^3}{(2-\beta)x - \frac{4}{3}x^3} = 3$$

$$\gamma - 1, \beta = 2, \frac{-3\alpha}{4} = +3 \Rightarrow \alpha = -4$$

$\beta + \gamma - \alpha = 7$

10. Ans (B)

$$10\frac{d}{dx}\int_1^x f(t)\,dt = \frac{d}{dx}\left(5xf(x) - x^5 - 9\right)$$

$\Rightarrow 10f(x) = 5f(x) + 5x\,f'(x) - 5x^4$

$\Rightarrow f(x) + x^4 = x\,f'(x)$

$$\Rightarrow y + x^4 = x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} + y\left(-\frac{1}{x}\right) = x^3$$

$$\Rightarrow ye^{\int -\frac{1}{x}dx} = \int x^3 e^{\int -\frac{1}{x}dx} + c$$

$$\Rightarrow ye^{-\ell n|x|} = \int x^3 e^{-\ell n|x|} + c$$

$$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$$

$$\Rightarrow \frac{y}{x} = \int x^2 + c$$

$$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$$

Put x = 1 in given equation

$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$

$$\Rightarrow \frac{2}{1} = \frac{1}{3} + c \Rightarrow c = \frac{5}{3}$$

$$\Rightarrow \frac{f(3)}{3} = \frac{27}{3} + \frac{5}{3}$$

$\Rightarrow f(3) = 32$

11. **Ans (A)**

$x + |x| > 0 \quad \Rightarrow x \in (0, \infty) \quad ....(1)$

$\& \ 10 + 3x - x^2 > 0$

$\Rightarrow x^2 - 3x - 10 < 0$

$\Rightarrow x \in (-2, 5) \quad ....(2)$

from (1) & (2) $\quad x \in (0, 5)$

$\therefore a = 0 \ \& \ b = 5$

$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$

12. **Ans (B)**

$$\int_0^1 \frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\,dx$$

$$-\,3\ell n\sqrt{3}$$

$$= 4\int_0^1 \frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{(3+x^2)-(1-x^2)}\,dx - \frac{3}{2}\ \ell n\,3$$

$$= 2\left[\left\{\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}\ln\left(x+\sqrt{3+x^2}\right)\right\}_0^1\right.$$

$$\left.-\left\{\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\ln(x+\sqrt{1+x^2})\right\}_0^1\right] - \frac{3}{2}\ \ell n3$$

$$= 2\left[\left\{\frac{1}{2}\sqrt{4}+\frac{3}{2}\ln(1+\sqrt{4})\right\}-\left\{0+\frac{3}{2}\ln\sqrt{3}\right\}\right.$$

$$\left.-\left\{\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln(1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2}\ln 3$$

$$= 2\left[1+\frac{3}{2}\ln 3-\frac{3}{4}\ln 3-\frac{1}{\sqrt{2}}-\frac{1}{2}\ln(1+\sqrt{2})\right]-\frac{3}{2}\ \ell n3$$

$$= 2 + 3\ \ell n\,3 - \frac{3}{2}\ \ell n\,3 - \sqrt{2} - \ell n\,(1+\sqrt{2}) - \frac{3}{2}\ln 3$$

$$= 2 - \sqrt{2} - \ln(1+\sqrt{2})$$

13. **Ans (A)**

$$\lim_{x\to 0}\frac{\cos 2x + a\cos 4x - b}{x^4} = \text{finite}$$

$$L = \frac{\left\{1-\frac{(2x)^2}{\lfloor 2}+\frac{(2x)^4}{\lfloor 4}....\right\}+a\left\{1-\frac{(4x)^2}{\lfloor 2}+\frac{(4x)^4}{\lfloor 4}....\right\}-b}{x^4}$$

$$L = \frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6(\ )....}{x^4}$$

$\therefore 1 + a - b = 0$ and $2 + 8a = 0 \ \Rightarrow a = -\dfrac{1}{4}$

$b = a + 1$

$= -\dfrac{1}{4} + 1 = \dfrac{3}{4}$

$\therefore a + b = -\dfrac{1}{4} + \dfrac{3}{4} = \dfrac{1}{2}$

14. **Ans (B)**

Let angle be θ then replacing (x, y) by

$(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

then $9x^2 - 2\sqrt{3}xy + 7y^2 = 10$ becomes

$9(x\cos\theta - y\sin\theta)^2 - 2\sqrt{3}\,(x\cos\theta - y\sin\theta)$

$(x\sin\theta + y\cos\theta) + 7(x\sin\theta + y\cos\theta)^2 = 10$

$\Rightarrow x^2(9\cos^2\theta - \sin\theta\cos\theta + 7\sin^2\theta) + 2xy(-9\sin\theta$

$\cos\theta - \sqrt{3}\cos 2\theta + 7\sin\theta\cos\theta)$

$+ y^2(9\cos^2\theta + \sin\theta\cos\theta + 7\cos^2\theta) = 10$

On comparing with $3x^2 + 5y^2 = 5$

(coefficient of xy = 0)

We get $-9\sin\theta\cos\theta - \sqrt{3}\cos 2\theta + 7\sin\theta\cos\theta = 0$

or $\sin 2\theta = -\sqrt{3}\cos 2\theta$

or $\tan 2\theta = -\sqrt{3} = \tan(180° - 60°)$

or $2\theta = 120°$

$\therefore \quad \theta = 60°$

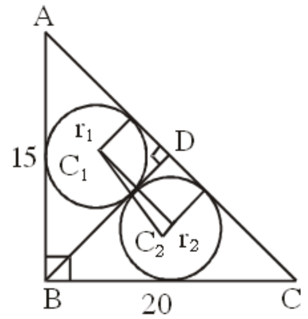
15. Ans (D)

$AC = \sqrt{15^2 + 20^2} = 25$

$BD = 12$

$AD = 9$

$CD = 16$



$$r_1 = \frac{\Delta}{S} = \frac{\frac{1}{2}\times 12 \times 9}{\frac{12+9+15}{2}} \Rightarrow \frac{9 \times 12}{36} = 3$$

$$r_2 = \frac{\Delta}{S} = \frac{\frac{1}{2}\times 16 \times 12}{\frac{20+16+12}{2}} \Rightarrow \frac{16 \times 12}{48} = 4$$

distance C_1C_2

$$= \sqrt{(r_2 - r_1)^2 + (r_1 + r_2)^2} = \sqrt{1+49} = \sqrt{50}$$

16. Ans (C)

$x^2 = 4y$

$x - \sqrt{2}y + 4\sqrt{2} = 0$

Solving together we get

$$x^2 = 4\left(\frac{x + 4\sqrt{2}}{\sqrt{2}}\right)$$

$\sqrt{2}x^2 + 4x + 16\sqrt{2}$

$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$

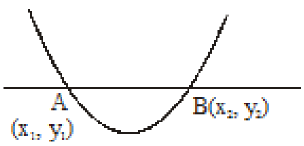
$$x_1 + x_2 = 2;\ \sqrt{2} \qquad x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$$

Similarly,

$\left(\sqrt{2}y - 4\sqrt{2}\right)^2 = 4y$

$2y^2 + 3^2 - 16y = 4y$

$2y2 - 20y + 32 = 0 \quad \begin{cases} y_1 + y_2 = 10 \\ y_1\, y_2 = 16 \end{cases}$



$$\ell_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(2\sqrt{2}\right)^2 + 64 + (10)^2 - 4(16)}$$

$$= \sqrt{8 + 64 + 100 - 64}$$

$$= \sqrt{108} = 6\sqrt{3}$$

17. Ans (B)

$a_1e_1 = a_2e_2$ or $a_1^2e_1^2 = a_2^2e_2^2$

or $\quad a_1^2\left(1 - \frac{b_1^2}{a_1^2}\right) = a_2^2\left(1 + \frac{b_2^2}{a_2^2}\right)$

or $\quad a_1^2 - b_1^2 = a_2^2 + b_2^2$

$$16 - a = \left(\frac{12}{5}\right)^2 + \left(\frac{9}{5}\right)^2 = \frac{225}{25} = 9$$

$\therefore \quad a = 7$

18. Ans (B)

Let side of the cube = a

Then OG, BE and AD, CF will be four diagonals.

d.r.'s of OG = a,a,a=1,1,1

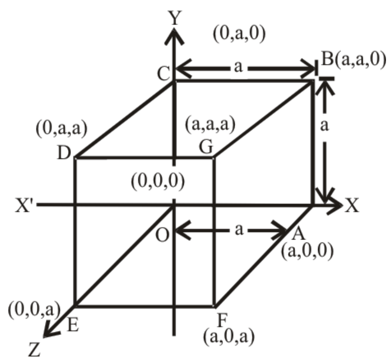
d.r.'s of BE = –a,–a,a=1,1,–1

d.r.'s of AD = –a,a,a= –1,1,1

d.r.'s of CF = a,–a,a=1,–1,1

Let d.r's of line be l, m, n.

Therefore angle between line and diagonal



$$\cos\alpha = \frac{l+m+n}{\sqrt{3}}, \cos\beta = \frac{l+m-n}{\sqrt{3}},$$

$$\cos\gamma = \frac{-l+m+n}{\sqrt{3}}, \cos\delta = \frac{l-m+n}{\sqrt{3}},$$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$=1/3[(l+m+n)^2+(l+m-n)^2+(-l+m+n)^2+(l-m+n)^2] = 4/3.$

19. **Ans (B)**

Varriance of observation $(x_1 - 8)$ $\forall$ i = 1,2,3,....18

$$= \frac{45}{18} - \left(\frac{9}{18}\right)^2 = \frac{5}{2} - \frac{1}{4} = \frac{9}{4}$$

then S.D. of x_1 $\forall$ i = 1, 2, 3, ... 18

$$= \sqrt{\frac{9}{4}} = \frac{3}{2}$$

20. **Ans (D)**

$\operatorname{cosec}^2 x + 25\sec^2 x$

$= 26 + \cot^2 x + 25\tan^2 x$

$= 26 + 10 + (\cot x - 5\tan x)^2 \geq 36$

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (2549)**

sum $= (5+4+2+1)\times 3\times 3! + (5+4+2+1)\,3\times 3!\times 10$

$+ (5+4+2+1)\,3\times 3!\times 100$

$+ (5+4+2+1)\,3\times 3!\times 1000$

$+ (5+4+2+1)\,4!\times 10000$

$= 216 + 2160 + 21600 + 216000 + 2880000$

$= 3119976$

2. **Ans (5)**

$3x + 2y - z = 4$ ......(1)

$x + y - z = 1$ ......(2)

$2x - \mu y + 2z = 3$ ......(3)

from (1) & (2) we get $8x + (4 - \mu)y = 11$

from (2) & (3) we get $4x + (2 - \mu)y = 2\lambda + 3$

Now for infinite solutions

$$\frac{8}{4} = \frac{4-\mu}{2-\mu} = \frac{11}{2\lambda+3}$$

$4 - 2\mu = 4 - \mu$ | $4\lambda + 6 = 11$

$\boxed{\mu = 0}$ | $4\lambda = 5$

$\boxed{\lambda = 5/4}$

$4(\lambda + \mu) = 5$

3. **Ans (36)**

$f''(x) = f(x)$

$\Rightarrow f'(x) \,.\, f''(x) = f'(x)\cdot f(x)$

$$\Rightarrow \frac{(f'(x))^2}{2} = \frac{(f(x))^2}{2} + C$$

$\Rightarrow (f'(x))^2 = (f(x))^2 + C'$

$f(0) = 0,\ f'(0) = 3 \Rightarrow C' = 9$

$\therefore\ (f'(x))^2 = (f(x))^2 + 9$

$f'(x) = \sqrt{(f(x))^2 + 9}$ $\because$ $f'(x) \geq 0$

$$\int \frac{dy}{\sqrt{y^2+9}} = \int dx \Rightarrow \ell n\left|y + \sqrt{y^2+9}\right| = x + C$$

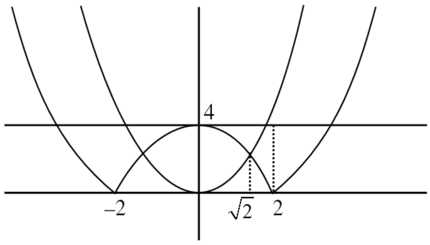
$\Rightarrow f(0) = 0 \Rightarrow C = \ell n3$

$\Rightarrow y + \sqrt{y^2 + 9} = 3e^x$

at $x = \ell n3$; $y = 4$

$\therefore\ 9f(\ell n3) = 36$

4. **Ans (22)**



$$A = \int_0^4 \sqrt{4+y}\,dy - \int_0^2 \sqrt{4-y}\,dy - \int_2^4 \sqrt{y}\,dy$$

$$= \left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 + \left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 - \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$\frac{80\sqrt{2}}{3} - 16 = \frac{40\sqrt{2}}{3} - 16$$

$\alpha = 6,\ \beta = 16$

$\alpha + \beta = 22$

5. **Ans (2)**

$$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$$

$$\vec{A} \times (\vec{R} \times \vec{B}) = \vec{A} \times (\vec{C} \times \vec{B})$$

$$\Rightarrow (\vec{A} \cdot \vec{B})\vec{R} - (\vec{A} \cdot \vec{R})\vec{B} = (\vec{A} \cdot \vec{B})\vec{C} - (\vec{A} \cdot \vec{C})\vec{B}$$

$$\Rightarrow (-38)\vec{R} - \vec{O} = (-38)\vec{C} - 19\vec{B}$$

$$\Rightarrow \frac{|\vec{B}|}{|\vec{R} - \vec{C}|} = 2$$