

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (MAIN)

Test Type : ALL INDIA OPEN TEST (MAJOR)

Test Pattern : JEE-Main

ANSWER KEY

| | | | | | | | | | | | | | | | | | | | | |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Ans. | 3 | 1 | 1 | 3 | 4 | 3 | 2 | 4 | 2 | 1 | 1 | 1 | 4 | 1 | 2 | 2 | 1 | 1 | 2 | 4 |
| Que. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Ans. | 3 | 4 | 1 | 3 | 1 | 1 | 3 | 3 | 3 | 2 | 4 | 1 | 1 | 3 | 3 | 4 | 2 | 3 | 1 | 2 |
| Que. | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| Ans. | 4 | 2 | 4 | 4 | 1 | 1 | 2 | 4 | 1 | 1 | 3 | 3 | 1 | 1 | 3 | 4 | 4 | 3 | 2 | 4 |
| Que. | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| Ans. | 3 | 2 | 4 | 4 | 1 | 4 | 1 | 2 | 1 | 4 | 2 | 2 | 1 | 3 | 1 | 4 | 1 | 2 | 1 | 1 |
| Que. | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | | | | | | | | | | |
| Ans. | 4 | 4 | 1 | 4 | 4 | 2 | 3 | 2 | 3 | 4 | | | | | | | | | | |

HINT - SHEET

SECTION-I

1. Ans. (3)

Sol. Force between two magnetic dipoles parallel to each.

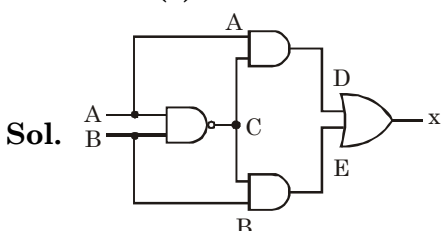
$$= \vec{F}_{ab} = \frac{3\mu_0}{2\pi r^4} (\vec{m}_a \vec{m}_b) \hat{r}$$

$$|\vec{F}| = mg$$

$$\therefore \frac{3\mu_0}{2\pi r^4} (m_b m_a) = mg$$

$$r = \left(\frac{3\mu_0 m^2}{2\pi mg} \right)^{1/4}$$

2. Ans. (1)



First we consider the output C of the NAND gate :

A B C

0 0 1

0 1 1

1 0 1

1 1 0

Next we consider the state D which is the output of an AND gate operating on A and C:

A B C D

0 0 1 0

0 1 1 0

1 0 1 1

1 1 0 0

Similarly, E is the output of an AND gate operating on B and C:

| | | | | |
|---|---|---|---|---|
| A | B | C | D | E |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |

Finally, the output X is the result of an OR gate operating on D and E:

| | | | | | |
|---|---|---|---|---|---|
| A | B | C | D | E | X |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |

We recognise this as the exclusive-or (XOR) function.

3. **Ans. (1)**

Sol. On displacing cylinder by x. Net force on cylinders

$$F_N = \rho_1 x s g + \rho_2 x s g = 2ma = (\rho_1 + \rho_2) x s g$$

$$\bar{a} = - \left(\frac{\rho_1 + \rho_2}{\rho_0} \right) \frac{s g}{2v} \bar{x}$$

$$\omega = \sqrt{\left(\frac{\rho_1 + \rho_2}{\rho_0} \right) \frac{s g}{2v}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\left(\frac{\rho_0}{\rho_1 + \rho_2} \right) \frac{2v}{s g}}$$

4. **Ans. (3)**

Sol. Reyleigh's criterion for the angular $\Delta\theta$ of a circular aperture of diameter $\Delta \gg \lambda$ gives

$$\Delta\theta = 1.22 \frac{\lambda}{D}$$

and for two objects at distance z separated by s the angular separation is

$$\Delta\theta = \frac{s}{z} \text{ (provided } z \ll z).$$

Combining these two expressions gives

$$D = 1.22 \frac{\lambda z}{s}$$

5. **Ans. (4)**

6. **Ans. (3)**

Sol. Force is developed due to property of liquid called surface tension.

7. **Ans. (2)**

8. **Ans. (4)**

Sol. $v_0 = \sqrt{\frac{GM}{R}}$

HS-2/8

For satellite launches with velocity $\frac{v_0}{2}$

$$TE = KE + PE$$

$$= -\frac{GMm}{R} + \frac{1}{2} m \left(\frac{v_0}{2} \right)^2$$

$$= -mv_0^2 + \frac{1}{8} mv_0^2 = -\frac{7}{8} mv_0^2$$

By energy conservation

$$(TE)_i = (TE)_f$$

$$-\frac{7}{8} mv_0^2 = -\frac{GMm}{R_1} + \frac{1}{2} mv_1^2$$

Since at lowest distance velocity is along tangential direction again.

$$\therefore L_i = L_f$$

$$\frac{mv_0 R}{2} = mv_1 R_1$$

$$v_1 = \frac{Rv_0}{2R_1}$$

$$-\frac{7}{8} v_0^2 = -\frac{Rv_0^2}{R_1} + \frac{1}{2} \left(\frac{Rv_0}{2R_1} \right)^2$$

$$-8RR_1 + R^2 = -7R_1^2$$

$$(7R_1 - R)(R_1 - R) = 0$$

$$R_1 = R_1 R/7$$

$$\therefore \text{Lowest point is } R_1 = R/7$$

9. **Ans. (2)**

Sol. By considering L - R circuit

$$I = I_0 e^{-\frac{Rt}{L}}$$

$$\text{Where } R = \frac{\rho(2\pi r)}{\ell d} \text{ and } L = \frac{\mu_0 \pi r^2}{\ell}$$

10. **Ans. (1)**

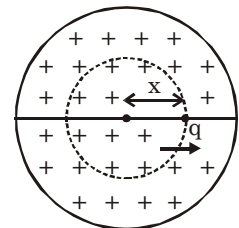
Sol. By dimensional analysis.

11. **Ans. (1)**

Sol. Charge q executes SHM along diameter about centre as mean position.

$$\vec{F}_q = \frac{-kqQ\vec{x}}{R^3}$$

$$T = 2\pi \sqrt{\frac{4\pi\epsilon_0 m R^3}{qQ}}$$



Time taken to move from centre to boundary

$$= \frac{T}{4} = \frac{\pi}{2} \sqrt{\frac{4\pi\epsilon_0 m R^3}{qQ}}$$

12. Ans. (1)

$$\text{Sol. } n_0 = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$$

$$4d^2\mu n_0^2 = T$$

frequency of oscillation is same as that of A.C.

13. Ans. (4)

Sol. Image distance is 30 cm

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f}$$

$$\frac{1}{30} = \frac{2}{f}$$

$$f = 60 \text{ cm}$$

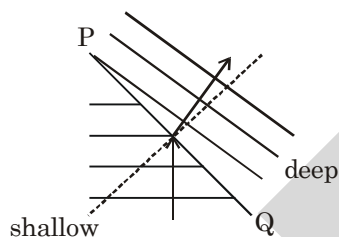
As optical axis shifts down by $(y - x)$ for second lens

$$m = \frac{y - x - 1}{y - x}$$

$$y = 4.5 \text{ cm}$$

14. Ans. (1)

Sol. In deep region wave bend towards normal and wavefront is perpendicular to wavefront.



15. Ans. (2)

$$\text{Sol. } V_{RL} = i_0 \times \sqrt{\omega^2 L^2 + R^2}$$

$$= \frac{V_0}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}} \times \sqrt{\omega^2 L^2 + R^2}$$

$$\omega = 0, \quad V_{RL} \propto V_0 \times \omega C$$

$$V_C = i_0 \times \frac{1}{\omega C}$$

$$\omega = \infty \quad V_{RL} = V_0$$

$$= \frac{V_0}{\sqrt{(\omega^2 LC - 1)^2 + \omega^2 R^2 C^2}}$$

$$\omega = 0$$

$$V_C = \frac{V_0}{\omega} \sqrt{LC}$$

$$\omega = \infty$$

16. Ans. (2)

$$\text{Sol. } f_B = \frac{c}{c+u} f_A < f_A$$

$$\text{Heard by A } f'_B = \frac{c-u}{c} f_B = \frac{c-u}{c+u} f_A < f_A$$

17. Ans. (1)

$$\text{Sol. } \frac{x}{2} = \frac{\ell}{1-\ell}$$

$$\frac{2}{x} = \frac{\ell+0.2}{1-(\ell+0.2)} = \frac{\ell+0.2}{0.8-\ell}$$

$$\frac{\ell(\ell+0.2)}{(1-\ell)(0.8-\ell)} = 1$$

$$\ell^2 + 0.2\ell = 0.8 + \ell^2 - 0.8\ell - \ell$$

$$2\ell = 0.8$$

$$\ell = 0.4 \text{ m}$$

$$\frac{x}{2} = \frac{0.4}{0.6} \Rightarrow x = \frac{4}{3}$$

$$\frac{2}{x} = \frac{\ell-0.2}{1-(\ell-0.2)} = \frac{\ell-0.2}{1.2-\ell}$$

$$1 = \frac{\ell(\ell-0.2)}{(1-\ell)(1.2-\ell)}$$

$$\ell^2 - 0.2\ell = 1.2 + \ell^2 - 2.2\ell$$

$$2\ell = 1.2$$

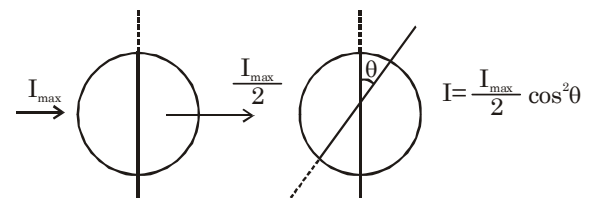
$$\ell = 0.6$$

$$\Rightarrow x = \frac{2 \times 0.6}{0.4} = 3\Omega$$

18. Ans. (1)

Sol. By Malus's law

$I = I_0 \cos^2 \theta$ θ is the angle between two axes of polaroid.



Third polaroid matches $(90-\theta)$ with IInd polaroid

$$I = \frac{I_{\max}}{2} \sin^2 2\theta \Rightarrow \frac{I_{\max}}{16} [1 - \cos 4\theta]$$

$$I = \frac{I_{\max}}{16} [1 - \cos 4\omega t]$$

19. Ans. (2)

Sol. $f = \frac{m_e q_e^4}{8h^3 \epsilon_0^2} \left(\frac{3}{4}\right) (z-1)^2$

$$\sqrt{f} = \sqrt{\frac{3}{4} \left(\frac{m_e q_e^4}{8h^3 \epsilon_0^2}\right)} (z-1)$$

$$a = \sqrt{\frac{3}{4} \frac{m_e q_e^4}{8h^3 \epsilon_0^2}} \text{ and } b = 1$$

Both a and b are constant and independent of target material.

20. Ans. (4)

Sol. With increase in frequency stopping potential increases in magnitude and with decreases in intensity saturation current decreases.

21. Ans. (3)

22. Ans. (4)

Sol. $y = mx + C$

$$\ln A = 2.6 - \frac{t}{10}$$

$$A = e^{2.6} e^{-t/10} = 12e^{-t/10}$$

23. Ans. (1)

24. Ans. (3)

Sol. $\phi = \vec{B} \cdot \vec{A}$

$$\text{and, } E_{\text{induced}} = -\frac{d\phi}{dt} = -Bvd$$

$$\text{Also, } F = i\vec{dl} \times \vec{B}$$

$$\text{Now, } e = E - E_{\text{induced}}$$

$$e = E - Bvd$$

$$i_{\text{total}} = i + i_{\text{ind}}$$

$$i_{\text{total}} = \frac{E - Bvd}{R}$$

$$\text{Therefore } F = \left(\frac{E - Bvd}{R}\right) Bd$$

$$m \frac{dv}{dt} = \left(\frac{E - Bvd}{R}\right) Bd$$

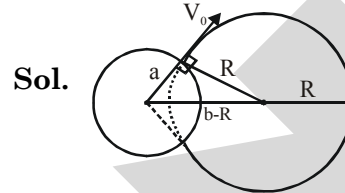
$$\frac{mR}{Bd} \int_0^v \frac{dv}{E - Bvd} = \int_0^t dt$$

$$-\frac{mR}{B^2 d^2} [\ln(E - Bvd)]_0^v = t$$

$$\ln\left(\frac{E - Bvd}{E}\right) = -\frac{B^2 d^2 t}{mR}$$

$$v = \frac{E}{Bd} \left(1 - e^{-\frac{B^2 d^2 t}{mR}}\right)$$

25. Ans. (1)



Pythagores theorem

$$a^2 + R^2 = (b - R)^2 = b^2 + R^2 - 2bR$$

$$\text{so, } \frac{mv_0}{qB} = R = \left(\frac{b^2 - a^2}{2b}\right)$$

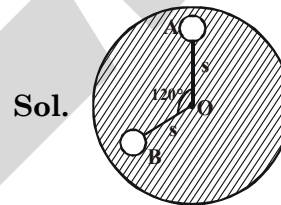
26. Ans. (1)

$$\text{Sol. } K_i = K_f + E_{\text{photon}} + K_{\text{atom}}$$

$$K_i - K_f = E_{\text{photon}} + K_{\text{atom}}$$

27. Ans. (3)

28. Ans. (3)



Sol.

$$\text{Initially } B_1 = \frac{\mu_0 I}{2\pi s}$$

Finally

$$B_2 = \sqrt{B_1^2 + B_1^2 + 2B_1 B_2 \cos 120^\circ}$$

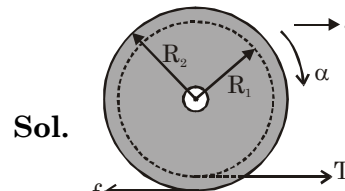
$$= 2B_1 \cos 60^\circ = B_1$$

So, B remains unchanged.

29. Ans. (3)

Sol. For narrow single slit diffraction width of central maxima is $\left(\frac{\lambda}{b}\right)$. where, $\lambda = \frac{h}{mv}$ and b is slit width.

30. Ans. (2)



Sol.

Force equation

$$T - f = ma \quad \dots (i)$$

Torque equation

$$fR_2 - TR_1 = I\alpha \quad \dots (ii)$$

Condition for no slipping,

$$a - \alpha R_2 = 0 \quad \dots (iii)$$

Solving we have,

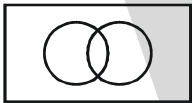
$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right) T$$

31. Ans. (4)
32. Ans. (1)
33. Ans. (1)
34. Ans. (3)
35. Ans. (3)
36. Ans. (4)
37. Ans. (2)
38. Ans. (3)
39. Ans. (1)
40. Ans. (2)
41. Ans. (4)
42. Ans. (2)
43. Ans. (4)
44. Ans. (4)
45. Ans. (1)
46. Ans. (1)
47. Ans. (2)
48. Ans. (4)
49. Ans. (1)
50. Ans. (1)
51. Ans. (3)
52. Ans. (3)
53. Ans. (1)
54. Ans. (1)
55. Ans. (3)
56. Ans. (4)
57. Ans. (4)
58. Ans. (3)
59. Ans. (2)
60. Ans. (4)
61. Ans. (3)

Sol. Mode = $l + \frac{(f_0 - f_1)}{(2f_0 - f_1 - f_2)} \times h$
 $h = 5; f_0 = 11, f_1 = f_2 = 7, f_2 = 2$

62. Ans. (2)

Sol. $A - B = A \cap B^c$



$A - (A \cap B^c) = A \cap B$

63. Ans. (4)

Sol. Equation becomes

$4a^2 + 6ax - (x^4 + x^3 - 2x^2) = 0$
 $a = \frac{-6x \pm \sqrt{36x^2 + 16(x^4 + x^3 - 2x^2)}}{8}$

$a = \frac{-x^2}{2} - x$ and $a = \frac{x^2}{2} - \frac{x}{2}$

$x^2 + 2x + 2a = 0$ or $x^2 - x - 2a = 0$

$D_1 \geq 0$ & $D_2 \geq 0$

$a \in \left[-\frac{1}{8}, \frac{1}{2}\right]$

64. Ans. (4)

Sol. $\Delta = \frac{1}{2}(2ae)(b \sin \theta)$

Maximum when $\sin \theta = 1$

$\Delta = abe$

65. Ans. (1)

Sol. Let $f(x) = a(x - x_1)(x - x_2) \dots (x - x_{10})$

$\ln f(x) = \ln a + \ln(x - x_1) + \ln(x - x_2) + \dots + \ln(x - x_{10})$

$\frac{f(x)}{f'(x)} = \frac{1}{(x - x_1)} + \frac{1}{(x - x_2)} + \dots + \frac{1}{(x - x_{10})}$

$\frac{f(x)f''(x) - (f'(x))^2}{(f'(x))^2} = 0$

$= -\left(\frac{1}{(x - x_1)^2} + \frac{1}{(x - x_2)^2} + \dots + \frac{1}{(x - x_{10})^2}\right)$

66. Ans. (4)

Sol. $C_1 \rightarrow aC_1, C_2 \rightarrow bC_2$ & $C_3 \rightarrow cC_3$

$\frac{1}{abc} \begin{vmatrix} a(b^2 + c^2) & ab^2 & ac^2 \\ a^2b & b(c^2 + a^2) & bc^2 \\ a^2c & b^2c & c(a^2 + b^2) \end{vmatrix}$

$= \begin{vmatrix} b^2 + c^2 & b^2 & c^2 \\ a^2 & c^2 + a^2 & c^2 \\ a^2 & b^2 & a^2 + b^2 \end{vmatrix}$

Use $C_1 > (C_1 + C_2 + C_3)$ and solve

67. Ans. (1)

Sol. It is 0° form

Let $y = (\sin \theta)^{(\sin \theta - \sin^2 \theta)}$

$\ln y = (1 - \sin \theta)(\sin \theta \ln \sin \theta)$

$\ln y = \lim_{\theta \rightarrow 0} \frac{\ln \sin \theta}{\cos \operatorname{cosec} \theta}$ (Use L'Hopital and solve)

68. Ans. (2)

Sol. Let $z = x + iy$

$\sqrt{x^2 + y^2} + x + iy - 3x + 3iy = 0$

$y = 0$ and $\sqrt{x^2 + y^2} = 2x$

$x = 0$

(0, 0) is only possibility

69. Ans. (1)

Sol. $T_{(r+1)} = {}^{400}C_r 3^{\left(\frac{50-r}{8}\right)} 5^{\frac{r}{3}}$

$r = 0, 24, \dots, 24 \times 16$

Total terms = 17

70. Ans. (4)

Sol. $\sin^{-1}\left(1 - \frac{1}{(2+x^2)}\right)$

$y = \sin^{-1}(1)$ when $x \rightarrow \infty = \frac{\pi}{2}$

When $x = 0 \Rightarrow y = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

71. Ans. (2)

Sol. $\frac{dy}{dx} = \frac{\alpha_1}{\sqrt{1-x^2}} + \frac{\alpha_3(3\sin^{-1}x)^2}{\sqrt{1-x^2}} + \dots$
 $+ \frac{(2x+1)(\sin^{-1}x)^{2n}}{\sqrt{1-x^2}} + \frac{1}{(1+x^2)}$

$\frac{dy}{dx} > 0$ and function is one one

but range \neq codomain

\Rightarrow into function

72. Ans. (2)

Sol. $\int_1^{e^2} \frac{\tan^{-1}t}{t} dt - \int_1^{1/e^2} \frac{\tan^{-1}t}{t} dt$

$\left(\text{Put } t = \frac{1}{z} \Rightarrow dt = -\frac{dz}{z^2}\right)$

$\int_1^{e^2} \frac{\tan^{-1}t}{t} dt + \int_1^{1/e^2} \frac{\tan^{-1}\left(\frac{1}{z}\right)}{z} dz$

$= \int_1^{e^2} \frac{\pi dz}{2z} = \frac{\pi}{2}(2) = \pi$

73. Ans. (1)

Sol. Let $y^2 = t \Rightarrow 2y \frac{dy}{dx} = \frac{dt}{dx}$

$2 \frac{dt}{dx} = \frac{1}{x(x^2 \sin t + 1)}$

$\frac{dx}{dt} = \frac{x^3 \sin t + x}{2}$

$x^{-3} \frac{dx}{dt} = \frac{\sin t}{2} + \frac{x^{-2}}{2}$

Let $x^{-2} = z \Rightarrow x^{-3} \frac{dx}{dt} = -\frac{dz}{2dt}$

$-\frac{1}{2} \frac{dz}{dt} = \frac{\sin t}{2} + \frac{z}{2}$

$\frac{dz}{dt} = -\sin t - z$

$\frac{dz}{dt} + z = -\sin t$

$ze^t = -\int e^t \sin t dt$

$ze^t = -\frac{(-e^t \cos t + e^t \sin t)}{2} + C$

$\frac{e^{y^2}}{x^2} = e^t \frac{(\cos t - \sin t)}{2} + C$

$e^{y^2} \left(\frac{1}{x^2} - \frac{\cos t}{2} + \frac{\sin t}{2}\right) \pm C$

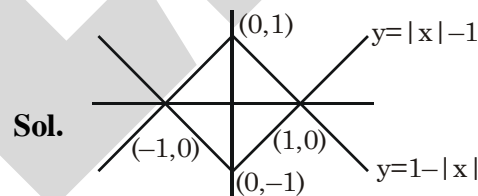
$e^{y^2} \left(\frac{1}{x^2} - \frac{\cos y^2}{2} + \frac{\sin y^2}{2}\right) = C$

74. Ans. (3)

Sol. $3\cos\theta + 4\sin\theta = 5$ & $3\sin\theta - 4\cos\theta = x$

Square and add $x^2 = 0$

75. Ans. (1)



Sol.

Area bounded = $(\sqrt{2})^2 = 2$

76. Ans. (4)

Sol. $f'(x)e^x + e^x f(x) \leq e^x$

$d(f(x)e^x) \leq e^x dx$

$\int_0^1 D(f(x)e^x) \leq \int_0^1 e^x dx$

$ef(1) \leq (e - 1)$

$f(1) \leq \frac{(e-1)}{e}$

77. Ans. (1)

Sol. $x = y^{x^x} \Rightarrow \ln x = x^x \ln y$ (when $x = 1, y = 1$)

$\frac{1}{x} = x^x (1 + \ln x) \ln y + \frac{x^x}{y} y'$

$y' = 1$

78. Ans. (2)

Sol. $ABC = I$

$$BC = A^{-1}$$

$$BCA = I$$

$$BC = A^{-1}$$

$$C = B^{-1}A^{-1}$$

$$CA = B^{-1} \Rightarrow CAB = I$$

$$\text{tr}(3I) = 9$$

79. Ans. (1)

Sol. $\left(\cos^2 2x - \cos x \cos^2 5x + \cos^4 \frac{5x}{4}\right) + \frac{\cos^2 5x(1 - \cos^2 5x)}{4} = 0$

$$\cos 2x = \frac{\cos^2 5x}{2} \quad \& \quad \sin 10x = 0$$

$$x = \frac{n\pi}{10}$$

$$x = 0, \frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10}$$

No solution

80. Ans. (1)

Sol. $V = \pi r^2 h$

$$V = \pi r^2(6 - r^2)$$

$$V = \pi(6r^2 - r^4)$$

$$\frac{dV}{dr} = \pi(12r - 4r^3) = 0$$

$$r^2 = 3h = 3$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

81. Ans. (4)

Sol. $\frac{dy}{dx} = 2(Ae^{2x} - Be^{-2x})$

$$\frac{d^2y}{dx^2} = 4(Ae^{2x} + Be^{-2x}) \Rightarrow \frac{d^2y}{dx^2} = 4y$$

82. Ans. (4)

Sol. $\int_0^1 \tan^{-1}\left(\frac{\tan x}{2}\right) dx - \int_0^1 \tan^{-1}(\cot x) dx$

$$\alpha - \int_0^1 \left(\frac{\pi}{2} - x\right) dx$$

$$\alpha - \frac{\pi}{2} + \frac{1}{2}$$

83. Ans. (1)

Sol. Sample space = 36

$A > B$ then

$$(6,1) (6,2) (6,3) (6,4) (6,5)$$

$$(5,1) (5,2) (5,3) (5,4)$$

$$(4,1) (4,2) (4,3) (3,1) (3,2) \& (2,1)$$

$$\text{Probability} = \frac{15}{36} = \frac{5}{12}$$

84. Ans. (4)

Sol. $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y' = 0$

$$\Rightarrow y' = -\frac{\sqrt{y}}{\sqrt{x}} \Rightarrow y' = -\frac{1}{2}$$

$$\text{Length of subtangent} = \left| \frac{y}{dy/dx} \right| = 2$$

85. Ans. (4)

Sol. $R_2 \rightarrow R_2 - 2R_1$ & $R_3 \rightarrow R_3 - 3R_1$

$$\begin{vmatrix} 1 & x & y \\ 0 & \sin x & \sin y \\ 0 & \cos x & \cos y \end{vmatrix} = \sin(x - y)$$

86. Ans. (2)

Sol. $\left(A - \frac{I}{2}\right)\left(A^T - \frac{I}{2}\right) = I$

$$AA^T - \frac{A^T}{2} - \frac{A}{2} = \frac{3I}{4} \quad \dots(1)$$

$$\text{Similarly } AA^T + \frac{A^T}{2} + \frac{A}{2} = \frac{3I}{4} \quad \dots(2)$$

$$(2) - (1) \Rightarrow A + A^T = 0$$

Skew symmetric matrix

But (1) + (2)

$$\Rightarrow AA^T = \frac{3I}{4}$$

But $|A| \neq 0$

87. Ans. (3)

Sol. $xy = c^2$

$$2xy = 2c^2$$

$$\frac{xc}{t} + (ct)y = 2c^2 \text{ (tangent)}$$

$$\frac{x}{t} + ty = 2c$$

$$a_1 = 2ct, \quad b_1 = \frac{2c}{t}$$

for normal

$$y - \frac{c}{t} = t^2(x - ct)$$

$$a_2 = \left(ct - \frac{c}{t^3} \right) \text{ \& } b_2 = \left(\frac{c}{t} - ct^3 \right)$$

$$a_1 a_2 + b_1 b_2 = 0$$

88. **Ans. (2)**

Sol.
$$[\vec{a} \ \vec{b} \ \vec{c}]^2 = \begin{vmatrix} \hat{a} \cdot \hat{a} & \hat{a} \cdot \hat{b} & \hat{a} \cdot \hat{c} \\ \hat{b} \cdot \hat{a} & \hat{b} \cdot \hat{b} & \hat{b} \cdot \hat{c} \\ \hat{c} \cdot \hat{a} & \hat{c} \cdot \hat{b} & \hat{c} \cdot \hat{c} \end{vmatrix}$$

$$[\vec{a} \ \vec{b} \ \vec{c}]^2 = \begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$$

$$[\vec{a} \ \vec{b} \ \vec{c}]^2 = (1 - \cos\theta)^2(1 + 2\cos\theta) \geq 0$$

$$\cos\theta \geq -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

89. **Ans. (3)**

Sol. $y' = \frac{1 - \ln x}{x^2} \text{ \& } y' = 2\lambda x$

$$1 - \ln x = 2\lambda x^3$$

$$\ln x = \lambda x^3$$

$$\ln x = \frac{1}{3} \text{ \& } x = e^{\frac{1}{3}}$$

$$\frac{1}{3} = \lambda e$$

$$\lambda = \frac{1}{3e}$$

90. **Ans. (4)**

Sol. $(x\vec{a} + y\vec{b} + z\vec{c}) \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$
 $= 2(x + y + z)$

$$(x + y + z)[\vec{a} \ \vec{b} \ \vec{c}] = 2(x + y + z)$$