NURTURE TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced)

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

PAPER-1

PART-1 : PHYSICS   **ANSWER KEY**

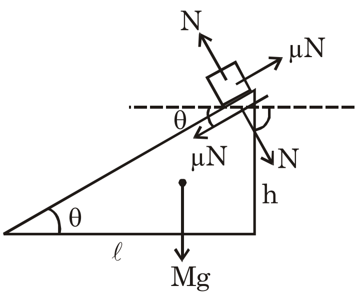
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	B,C	B,C	A,C	B,C,D	A,B,D	A,D	A	C	D
	Q.	11	12	13							
	A.	B	C	D							
SECTION-IV	Q.	1	2	3	4	5					
	A.	2	5	6	6	5					

SOLUTION

SECTION-I

1. Ans. (A,C)

Sol.



$$N\cos(90-\theta)h - \mu N\cos\theta h - Mg\cdot\frac{\ell}{3} = 0$$

$$(N\sin\theta - \mu N\cos\theta)h = Mg\frac{\ell}{3}$$

$$(mg\cos\theta)(\sin\theta-\mu\cos\theta) = \frac{Mg}{3\tan\theta}$$

$$m = \frac{M}{3\tan\theta\cdot\cos\theta(\sin\theta-\mu\cos\theta)}$$

$$m = \frac{M}{3\sin\theta(\sin\theta-\mu\cos\theta)}$$

2. Ans. (B,C)

Sol. $$\frac{F}{2\pi x\ell} = \eta\frac{dv}{dx}$$

$$\frac{F}{2\pi\ell}\int_{R}^{2R}\frac{dx}{x} = \eta\int_{-v_0}^{2v_0} dv$$

$$\frac{F}{2\pi\ell}\times \ell n2 = \eta\times 3v_0$$

$$\frac{F}{\ell} = \frac{6\pi v_0\eta}{\ell n2}$$

$$\frac{F}{2\pi\ell}\times \ell n\left(\frac{x}{R}\right) = \eta v_0$$

$$\Rightarrow \frac{3}{\ell n2}\ell n\left(\frac{x}{R}\right) = 1$$

$$\Rightarrow \ell n\left(\frac{x}{R}\right) = \ell n2^{1/3} \Rightarrow x = 2^{1/3}R$$

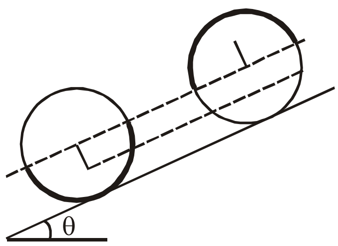
3. Ans. (B,C)

Sol. If the helicopter is at terminal velocity, then it is falling at constant speed. As such, h = vT, where v is the terminal velocity. In that case, α = 1.

Dimensional analysis on mass required that δ = –ω. Since only W has units of time (inverse squared), then ω = –1/2. But $\sqrt{\rho/W}$ has units of length^{-2} time^{-1}, and we known α = 1, then β = 1

4. Ans. (A,C)

Sol.



$$I_{cm} = I_0 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1-\frac{4}{\pi^2}\right)$$

$$U = mg\left[\frac{4r}{\pi}\cos\theta + \pi r\sin\theta\right]$$

$$U = mgr\left[\frac{4}{\pi}\cos\theta + \pi\sin\theta\right] = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$mgr\left[\frac{4}{\pi}\cos\theta + \pi\sin\theta\right]$$

$$= \frac{1}{2}m\left[\omega\left(r-\frac{2r}{\pi}\right)\right]^2 + \frac{1}{2}mr^2\left(1-\frac{4}{\pi^2}\right)\omega^2$$

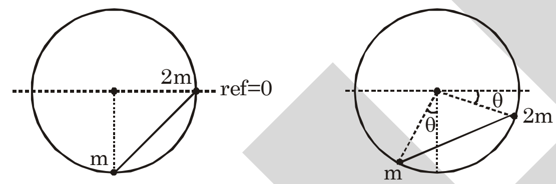
$$= m\omega^2r^2\left(\frac{\pi-2}{\pi}\right)$$

$$\omega = \sqrt{\frac{g}{r}\left(\frac{4\cos\theta+\pi^2\sin\theta}{\pi-2}\right)}$$

5. Ans. (B,C,D)

6. Ans. (A,B,D)

Sol.



$u_i = -mgr$ $\quad$ $v_f = -2mgr\sin\theta - mgr\cos\theta$

$\Delta u = u_f - u_i$

$= mgr - (2mgr\sin\theta + mgr\cos\theta)$

$$-\frac{1}{2}(3m)v^2 = mgr[1-2\sin\theta-\cos\theta]$$

$$v^2 = \frac{2}{3}gr[-1+2\sin\theta+\cos\theta]$$

for v_{max}

$$\Rightarrow \frac{d(v^2)}{d\theta} = 0 \Rightarrow \frac{2}{3}[0-2\cos\theta+\sin\theta] = 0$$

$\tan\theta = 2$

for $\theta = \theta_{max}$, $v = 0$

7. Ans. (A,D)

8. Ans. (A)

Sol. $\omega_{AB} = 0$, $\Delta U_{BC} = 3nRT_0$, $\eta = \frac{2}{13}$

9. Ans. (C)

10. Ans. (D)

11. Ans. (B)

Sol. $y = A\sin\left(2\pi\nu t - \frac{2\pi}{v}\nu x + \phi\right)$

$$\frac{A}{2} = A\sin\left(0-\frac{2\pi}{3}+\phi\right)$$

$$\Rightarrow \frac{\pi}{6} = -\frac{2\pi}{3}+\phi$$

$$\phi = \frac{\pi}{6}+\frac{2\pi}{3}$$

$$\phi = \frac{5\pi}{6}$$

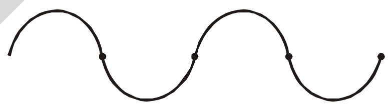
$$y = A\sin\left(2\pi\nu t - \frac{2\pi}{v}\nu x + \frac{5\pi}{6}\right)$$

$$E = \frac{\mu A^2\omega^2}{2}\times\lambda = 2\pi^2\mu A^2\nu^2\lambda$$

$$P = \frac{\mu v A^2\omega^2}{2} = 2\pi^2\mu A^2\nu^2 v$$

12. Ans. (C)

Sol.



$$y = A\sin(kx)\cos(\omega t) = A\sin\left(\frac{2\pi}{\ell}\times 2x\right)\cos(\omega t)$$

$$\ell = 2\lambda \Rightarrow \lambda = \frac{\ell}{2}$$

$$y = A\sin\left(\frac{4\pi}{\ell}x\right)\cos(\omega t)$$

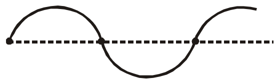
$$E = \int_0^\lambda \frac{1}{2}(dm)(\omega^2)\left(A^2\sin^2\left(\frac{4\pi}{\ell}x\right)\right)$$

$$= \frac{1}{2}\mu\omega^2A^2\int_0^\lambda \sin^2\left(\frac{4\pi}{\ell}x\right)dx$$

$$= \frac{1}{2}\mu\omega^2A^2\left[\frac{1}{2}\int_0^\lambda 1-\cos\left(\frac{8\pi x}{\ell}\right)dx\right]$$

$$= \frac{1}{2}\mu\omega^2A^2\frac{\lambda}{2} = \frac{\mu\omega^2A^2\lambda}{4}$$

13. Ans. (D)

Sol.  $\ell = \frac{5\lambda}{4}$

$y = A\sin(kx)\cos(\omega t)$

$$y = A\sin\left(\frac{2\pi}{4\ell} \times 5x\right)\cos(\omega t)$$

$$= A\sin\left(\frac{5\pi}{2\ell}x\right)\cos(\omega t)$$

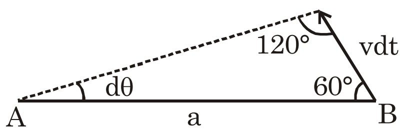
$$E = \int \frac{1}{2}(\mu dx)\omega^2 A^2 \cos^2\left(\frac{5\pi}{2\ell}x\right)$$

$$= \frac{1}{2}\mu\omega^2 A^2 \int_0^{\lambda} \cos^2\left(\frac{5\pi}{2\ell}x\right)dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \frac{1}{2}\cdot\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

SECTION-IV

1. Ans. 2

Sol. 

$$\frac{a}{\sin 120°} = \frac{vdt}{\sin(d\theta)} \simeq \frac{vdt}{d\theta}$$

$$\frac{2}{\sqrt{3}}a = \frac{v}{\omega}$$

$$a_C = v\omega = \frac{v\cdot\sqrt{3}v}{2a} = \frac{\sqrt{3}}{2}\frac{v^2}{a}$$

2. Ans. 5

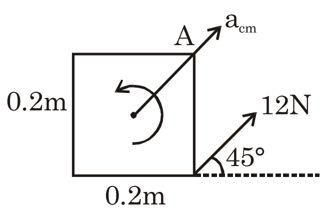
Sol. $F = \left(\frac{10}{2}\right)(10)^2\left(\frac{3\times 0.8}{8}\right) = 150N$

3. Ans. 6

Sol. $\sin\theta_m = \frac{m_1}{m_2} = \frac{1}{2}$

$\theta_m = 30°$

4. Ans. 6

Sol. 

$12 = 6a_{cm} \Rightarrow a_{cm} = 2$

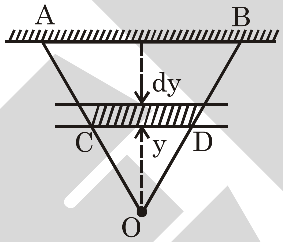
$$12\cdot\frac{\ell}{\sqrt{2}} = \frac{1}{6}\times 6(\ell^2)\alpha \Rightarrow \alpha = \frac{6\sqrt{2}}{\ell}$$

$$\vec{a}_A = \vec{a}_{A/cm} + \vec{a}_{cm/g}$$

$$|\vec{a}_A| = \sqrt{a_{cm}^2 + \left(\frac{\alpha\ell}{\sqrt{2}}\right)^2} = \sqrt{2^2+6^2} = \sqrt{40}$$

$$= 2\sqrt{10}\ \text{m/s}^2$$

5. Ans. 5

Sol. 

Weight of OCD = $m\left(\frac{y}{\ell}\right)^3 g$

Stress at CD = $\dfrac{m\left(\frac{y}{\ell}\right)^3 g}{\pi\left(\frac{y}{\ell}R\right)^2}$

$$\frac{\text{Elastic potential energy}}{\text{Volume}}$$

$$= \frac{1}{2}\frac{1}{Y}\left[\frac{m\left(\frac{y}{\ell}\right)^3 g}{\pi\left(\frac{y}{\ell}R\right)^2}\right]^2$$

Total elastic energy

$$= \frac{1}{2}\frac{1}{Y}\int_0^{\ell}\left[\frac{mg}{\pi R^2}\left(\frac{y}{\ell}\right)\right]^2\cdot\pi\left(\frac{y}{\ell}R\right)^2 dy = \frac{m^2g^2\ell}{10\pi R^2 Y}$$

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C,D	A,B,D	A,B,C	A,B	A,B,C,D	A,B,C,D	A,B,C,D	A	C	B
	Q.	11	12	13							
	A.	A	B	C							
SECTION-IV	Q.	1	2	3	4	5					
	A.	1	2	3	6	5					

SOLUTION

SECTION-I

1. **Ans. (C,D)**
2. **Ans. (A,B,D)**
3. **Ans. (ABC)**
4. **Ans. (A,B)**
5. **Ans. (A,B,C,D)**
6. **Ans. (A,B,C,D)**
7. **Ans. (A,B,C,D)**
8. **Ans. (A)**
9. **Ans. (C)**
10. **Ans. (B)**
11. **Ans. (A)**
12. **Ans. (B)**
13. **Ans. (C)**

SECTION-IV

1. **Ans. (1)**
2. **Ans. (2)**
3. **Ans. (3)**
4. **Ans. (6)**
5. **Ans. (5)**

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,B,C,D	B	A,C	A,C	C,D	C,D	C	B	D
	Q.	11	12	13							
	A.	D	C	B							
SECTION-IV	Q.	1	2	3	4	5					
	A.	0	3	6	4	2					

SOLUTION

SECTION-I

1. **Ans. (A,C)**

We must have

$(p-1)(q+2) = 0$ & $(p-1)(p^2+p-1) = 0$

$\Rightarrow p = 1$

Now, $g(x) = -4x^2 + (2q+1)x - 5$

$\therefore$ maximum value occur at $x = \frac{2q+1}{8}$

$\therefore \frac{(2q+1)^2}{16} - 5 = -1 \Rightarrow q = \frac{7}{2}$ or $\frac{-9}{2}$

$\Rightarrow (p+q)$ can be $\frac{9}{2}$ or $\frac{-7}{2}$

2. **Ans. (A,B,C,D)**

The given equation can be written as

$|x-1| = 4-\lambda$ or $|x-1| = -4-\lambda$

Now interpret.

3. **Ans. (B)**

$2[\alpha] + 64 = 3[\alpha] - 192$

$\Rightarrow [\alpha] = 256 \Rightarrow \alpha \in [256, 257)$

Also, $y = \frac{1}{4}(\sin 10°.\sin 50°.\sin 70°)^2$

$= \frac{1}{4}\left(\frac{1}{4}\sin 30°\right)^2$

$\Rightarrow \quad y = \frac{1}{256}$

4. **Ans. (A,C)**

Domain of $f(x) = x \in (-1, 1)$

Put $\sin^{-1}x = t$; $t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(t) = 4\left(t - \frac{\pi}{4}\right)^2 + \pi^2$

$\therefore f_{min.}\left(t = \frac{\pi}{4}\right) = \pi^2$

But $f_{max.}$ = does not exist

5. **Ans. (A,C)**

Clearly, power of point T(3,4) w.r.t. S = 0 is $S_1 = 14$.

Also, angle b/w tangents from T(3,4) to circle S = 0, is

$$= 2\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

The equation of circumcircle of ΔTAB is

$x(x - 3) + (y - 1)(y - 4) = 0$

$\Rightarrow \quad x^2 + y^2 - 3x - 5y + 4 = 0$

and area of quadrilateral TACB

$$= 2\left(\frac{1}{2}\right)(2)\sqrt{14} = 2\sqrt{14} \text{ (square units)}$$

6. Ans. (C,D)

Given a = 10, $a_{a_2} = 100$

Let d be the common difference of A.P.

$\therefore\ a_{a_2} = a + (a_2 - 1)d = 10 + (a_2 - 1)d$

$\Rightarrow 100 = 10 + (10 + d - 1)d$

$\Rightarrow \quad d = 6$

Now, $a_3 = a + 2d = 10 + 2(6) = 22$

$\therefore\ a_{a_3} = a_{22} = a + 21d$

$= 10 + 21(6) = 10 + 126 = 136$

7. Ans. (C,D)

Number of linear arrangements of 4 alike of one kind and 5 alike of another kind is equal to = $\frac{\lfloor 9}{\lfloor 4 \lfloor 5} = {}^9C_4 = 126$ ways.

(A) 126

(B) 126

(C) ${}^9C_4(2!) = 252$

(D) ${}^8C_1 + {}^8C_2 + {}^8C_3 + \frac{{}^8C_4}{2}$

$= 8 + 28 + 56 + 35 = 127$

8. Ans. (C)

9. Ans. (B)

10. Ans. (D)

(I) $f(x) = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$

$D_f = x \in (-\infty,\infty)$; $R_f = [0,\pi)$

(II) $g(x) = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$;

$D_f = R - \left\{\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$; $R_f = \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(III) $h(x) = \pi\left(\frac{\sqrt{x+7} - 4}{x - 9}\right)$

$D_h = (-7,\infty) - \{9\}$

$R_h = \left(0, \frac{\pi}{4}\right] - \left\{\frac{\pi}{8}\right\}$

(IV) $k(x) = \frac{\pi}{\sqrt{2}}\left(\sin(|x|) + \cos(|x|)\right)$

$D_k = x \in (-\infty,\infty)$

$R_k = [-\pi,\pi]$

Now verify yourself.

11. Ans. (D)

12. Ans. (C)

13. Ans. (B)

(I) The circles are $x^2 + y^2 - 12x + 27 = 0$ and $x^2 + y^2 = 4$. No. of common tangents b/w them is 4. Clearly, the distance of common tangent from O(0,0) is 2.

(II) The circles are $x^2 + y^2 - 6x - 6y + 9 = 0$ & $x^2 + y^2 + 6x + 6y + 9 = 0$

Here, no. of common tangents b/w them are 4. Also distance of common tangent from O(0,0) is 3.

(III) $x^2 + y^2 - 10x - 10y + 25 = 0$

& $x^2 + y^2 + 6x + 2y - 15 = 0$

∴ The equation of common tangent at (1,2) is 4x + 2y = 10, whose distance from O(0,0) is $\sqrt{5}$. No. of common tangents are 3.

(IV) The circles are $x^2 + y^2 = 4$

& $x^2 + y^2 - 2x - 2y + 1 = 0$

The equation common tangents is x = 2 & y = 2.

No. of common tangents are 2.

Also, distance of common tangent from O(0,0) is 2.

SECTION-IV

1. Ans. 0

$f(x) = (x^2 - 10x + 16)(x^2 - 10x + 24) + 16$

$= (t + 16)(t + 24) + 16,$

Where $t = (x^2 - 10x)$

$\therefore\ y = t^2 + 40t + 400 = (t + 20)^2$

$\Rightarrow y_{min.}(t = -20) = 0$

2. Ans. 3

$$T_n = \cot^{-1}\left(4 + \frac{n(n+1)}{4}\right)$$

$$\therefore\ S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n} \tan^{-1}\left(\frac{\frac{1}{4}}{1 + \frac{n}{4}\left(\frac{n+1}{4}\right)}\right)$$

$$= \sum_{n=1}^{n}\left(\tan^{-1}\left(\frac{n+1}{4}\right) - \tan^{-1}\left(\frac{n}{4}\right)\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{1}{4}\right)$$

3. Ans. 6

Case I : $x \in (-\infty, -1) \Rightarrow x = -\left(2 + \sqrt{3}\right)$

Case II : $x \in [-1,0) \Rightarrow x = 1$ (Rejected)

Case III : $x \in [0, 1] \Rightarrow x = \left(2 - \sqrt{3}\right)$

Case IV : $x \in (1, \infty) \Rightarrow x = -1$ (Rejected)

$$\therefore\ \left(-\left(2 + \sqrt{3}\right)\right)^2 + \left(2 - \sqrt{3}\right)^2 = 14$$

4. Ans. 4

Given,

$(3x - y - 1) + b(4x - y - 2) = 0$

$\therefore$ Fixed point is (1,2)

Let line be

$(y - 2) = m(x - 1),\ \ m \in (-\infty, 0)$

$$\therefore\ \text{Area}(m) = \frac{8}{2} + \frac{1}{2}\left(\sqrt{-m} - \frac{2}{\sqrt{-m}}\right)^2,\ m < 0$$

$\Rightarrow A_{Least}(m = -2) = 4$

5. Ans. 2

$a = 5,\ b = 6,\ c = 7$

$$R = \frac{35}{4\sqrt{6}}$$

Also, $\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} = \frac{r}{4R}$

$$= \frac{\left(\frac{\Delta}{s}\right)}{\left(\frac{abc}{\Delta}\right)} = \frac{\Delta^2}{s(abc)} = \frac{4}{35}$$

Ans, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{5}$

So, $\sin^2\left(\frac{A+B}{2}\right) = \cos^2\frac{C}{2}$

$$= \frac{1 + \cos C}{2} = \frac{1 + \frac{1}{5}}{2} = \frac{6}{10} = \frac{3}{5}$$

NURTURE TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced) 2019

Test Type : **ALL INDIA OPEN TEST** (MAJOR) Test Pattern : JEE-Advanced

PAPER-2

PART-1 : PHYSICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	B	D	D	C	A	A,B,C,D	B,D	A,B,C
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,C,D	A,B	C,D	A,D	C	D	B	B		

SOLUTION

SECTION-I

1. **Ans. (D)**
2. **Ans. (A)**

Sol. $\frac{d}{v-u} = 60$

$v + u = \frac{d}{60}$

$v - u = \frac{d}{80}$

$2u = \frac{d(8-6)}{480}$

$u = \frac{d}{240}$

$t = \frac{d}{u}$

3. **Ans. (B)**

Sol. $1.9 \times \pi \times 100 \times t = 19 \times 80$

$t = \frac{8}{\pi}$

4. **Ans. (D)**

Sol. $k_{eq} = k_1 + k_2 = 4k$

$T = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$

$t = \frac{T}{4} \Rightarrow$ reaches mean position.

$\Rightarrow k(\ell + x) = 3k(\ell - x)$

$\ell + x = 3\ell - 3x$

$4x = 2\ell$

$x = \frac{\ell}{2}$

5. **Ans. (D)**

Sol. $20g = T$

$T \sin\beta = T' \sin\alpha$

$200 \times 0.6 = T' \sin\alpha$

$T' \cos\alpha + T\cos\beta = 250$

$T' \cos\alpha + 200 \times 0.8 = 250$

$T' \cos\alpha = 90$

$T' \sin\alpha = 120$

$\tan\alpha = \frac{4}{3}$

$\alpha = 53°$

$T' = 150$

6. **Ans. (C)**

Sol. $5 - \int Tdt = 1 \times v_2 \downarrow$

$-\int Tdt = 1 \times v_1 \uparrow$

$-2\int Tdt = 2v \uparrow$

$v_1 + v_2 = 2v = 0$

$5 - 4\int Tdt = 0$

$\int Tdt = \frac{5}{4}$

7. **Ans. (A)**
8. **Ans. (A,B,C,D)**

Sol. $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2} + 2mg(R-r)$

$= \frac{7}{10}mv^2 + 2m(R-r) = \frac{27}{10}mg(R-r)$

$x = \sqrt{\frac{27mg(R-r)}{5k}}$

$mg = \frac{mv^2}{R-r}$

9. Ans. (B,D)

Sol. $\Delta U_m = 900 \times 10 \times 100 = 900$ kJ

$\Delta U_{rope} = m \times 10 \times 50 = 100$ kJ

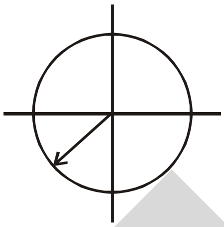
m = 200 kg

$\frac{m}{\ell} = 2kg/m$

10. Ans. (A,B,C)

11. Ans. (A,B,C,D)

Sol. $\theta = \frac{2\pi}{\frac{\pi}{5}} \times \frac{\pi}{3} = \frac{10\pi}{3}$



$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x_0}{g}} = \frac{\pi}{5}$

$\frac{2\pi}{3} = \omega t$

$\Rightarrow t = \frac{\pi}{15}\sec$

12. Ans. (A,B)

Sol. (A) $\Delta K = \frac{1}{2}\mu v^2_{rel}$

(B) $a_{mM} = a_m - a_M = -F\frac{(M+m)}{mM}$

$\Rightarrow \frac{-F}{m} = a_m \Rightarrow a_M$

(C) $0^2 = u^2 - 2d \times \frac{F(M+m)}{mM}$

$\Rightarrow F = \frac{mMu^2}{2d(M+m)}$

13. Ans. (C,D)

Sol. The curve with less wavelength as the frequency received by A, $v_A = \left(\frac{v}{v+vs}\right)v$

14. Ans. (A,D)

Sol. $\mu N \Rightarrow \delta = \frac{Mg}{2\ell^2}(R^2+\ell^2)$

$\frac{\mu MRg}{\ell}$

$\mu \geq \frac{(R^2+\ell^2)}{2\ell R}$

$\frac{\cos\theta}{2} = \frac{\ell}{\sqrt{R^2+\ell^2}}$

$T\sin\theta = N$

$N = mg\tan\frac{\theta}{2} = \frac{MgR}{\ell}$

$T = \delta$

$T\cos\theta + \delta = Mg$

$T = \frac{Mg}{\frac{2\cos^2\theta}{2}} = \frac{Mg}{2\ell^2}(R^2+\ell^2)$

15. Ans. (C)

16. Ans. (D)

17. Ans. (B)

Sol. $s = A_x\sin(\omega t + \phi)$

$k = \frac{\omega}{v}$

$A_x = A\sin(kx + \phi')$

$x = 0, A_x = s_0$

at $x = L$ $\frac{\partial A_x}{\partial x} = 0$

$\frac{\partial A_x}{\partial x} = Ak\cos(kL + \phi') = 0$

$\cos(kL + \phi') = 0$

$kL + \phi' = \frac{\pi}{2}$

$A_x = A\sin\left(k(x-L) + \frac{\pi}{2}\right)$

$= A[\cos kx \cos kL + \sin kx \sin kL]$

$= A\cos kx[\cos kL + \tan kx \sin kL]$

$x = 0, A_x = s_0$

$s_0 = A[\cos kL]$

$A = \frac{s_0}{\cos kL}$

$A_x = s_0\cos kx[1 + \tan kx \tan kL]$

18. Ans. (B)

Sol. $\omega = \frac{(2n+1)}{4L}\sqrt{\frac{Y}{\rho}} = \frac{10^4}{40}\times(2n+1)$

PART-2 : CHEMISTRY

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	A	B	A	C	B	C	A,B,C	B	A,B,C
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,D	A,B,D	A,B,C	A,D	B	D	A	D		

SOLUTION

SECTION-I

1. Ans. (C)
2. Ans. (A)
3. Ans. (B)
4. Ans. (A)
5. Ans. (C)
6. Ans. (B)
7. Ans. (C)
8. Ans. (A,B,C)
9. Ans. (B)
10. Ans. (A,B,C)
11. Ans. (A,B,D)
12. Ans. (A, B, D)
13. Ans. (A,B,C)
14. Ans. (A,D)
15. Ans. (B)
16. Ans. (D)
17. Ans. (A)
18. Ans. (D)

PART-3 : MATHEMATICS

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	B	D	B	B	A	B	A,C	B,C	A,B
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,C,D	A,C	A,B,C,D	B,D	D	C	A	D		

SOLUTION

SECTION-I

1. Ans. (C)

$$S_n = \sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+(r+1)(r+2)}\right)$$

$$S_n = \sum_{r=1}^{n}\left(\tan^{-1}(r+2) - \tan^{-1}(r+1)\right)$$

$$S_n = \tan^{-1}(n+2) - \tan^{-1}2$$

Now check options

2. Ans. (B)

$$f(x) = f\left(x + \frac{\pi}{2}\right)$$

$$f(x) = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4},$$

also $f\left(x+\frac{\pi}{4}\right) = f\left(\frac{\pi}{4} - x\right)$

3. Ans. (D)

$5[x] - [x]^2 > 0$

$[x]^2 - 5[x] < 0$

$0 < [x] < 5$

$[x] = 1, 2, 3, 4$

4. Ans. (B)

$$d(x,[0,1]) = \begin{cases} 0-x, & x<0 \\ 0, & 0 \le x \le 1 \\ x-1, & 1<x \end{cases}$$

$$d(x,[2,3]) = \begin{cases} 2-x, & x<2 \\ 0, & 2 \le x \le 3 \\ x-3, & 3<x \end{cases}$$

$$f(x) = \begin{cases} \dfrac{-x}{-x+2-x}, & x<0 \\ 0, & 0 \le x \le 1 \\ \dfrac{x-1}{x-1+2-x}, & 1<x<2 \\ 1, & 2 \le x \le 3 \\ \dfrac{x-1}{x-1+x-3}, & 3<x \end{cases}$$

Range $x \in [0,1]$ onto function

5. Ans. (B)

Let $b = ar$

$c = ar^2$

given $2\log_b c = \log_c a + \log_a b$

$$\Rightarrow 2\frac{\log_a ar^2}{\log_a ar} = \frac{1}{\log_a ar^2} + \log_a ar$$

$\Rightarrow \frac{2[1+2t]}{1+t} = \frac{1}{1+2t} + 1 + t$ {Let $\log_a r = t, t \neq 0$}

$\Rightarrow 2(1 + 2t)^2 = 1+t+(1 +2t)(1 + t)^2$

$\Rightarrow t(2t - 3t - 3) = 0$

$\Rightarrow 2t^2 - 3t - 3 = 0$

common difference $= \log_a b - \log_b c$

$= (1+t) - \left(\frac{1+2t}{1+t}\right) = \frac{t^2}{1+t} = \frac{3}{2}$

6. Ans. (A)

Let n(x) = a, X = {b + 1, b + 2,....,5}

n(y) = b, Y = {a + 1, a + 2,....,5}

number of ordered pairs

$= \sum_{1\le a,b\le 5} \binom{5-a}{b}\binom{5-b}{a} = 144$.

7. Ans. (B)

$V_{13}((2018)!) = \left\lfloor \frac{2018}{13} \right\rfloor + \left\lfloor \frac{2018}{13^2} \right\rfloor + \left\lfloor \frac{2018}{13^3} \right\rfloor + ...$

= 155 + 11 + 0

= 166

$\Rightarrow V_{13}(r!) + V_{13}((2018 - r)!) \le 165$

$\Rightarrow r = 13\lambda + k, k = 4,5,6,7,8,9,10,11,12$

for $r = 13\lambda + 4$ we have following 155 numbers

r = 4,17,30,....,2006

similiarly for others

$\Rightarrow$ Total $155 \times 9 = 1395$ values.

8. Ans. (A,C)

$$f(n,r) = \frac{1\binom{n-1}{r-1} + 2\binom{n-2}{r-1} + ... + (n-r+1)\binom{r-1}{r-1}}{\binom{n}{r}}$$

$$= \frac{\binom{n}{r} + \binom{n-1}{r} + \binom{n-2}{r} + ... + \binom{r}{r}}{\binom{n}{r}}$$

$$= \frac{\binom{n+1}{r+1}}{\binom{n}{r}} = \frac{n+1}{r+1}$$

9. Ans. (B,C)

$(x^5 - x^3 - 4x^2 - 3x - 2) + \lambda (5x^4 + \alpha x^2 - 8x + \alpha) = 0$

$\Rightarrow (x - 2)(x^2 + x + 1)^2 + \lambda(5x^4 + \alpha x^2 - 8x + \alpha) = 0$

A root is independent of λ if its is a common root of $(x - 2)(x^2 + x + 1)^2 = 0$ and $5x^4 + \alpha x^2 - 8x + \alpha = 0$

for $\alpha = -\frac{64}{5}$, x = 2 is a common root

for $\alpha = -3$, $x^2 + x + 1$ is common factor which correspond to two roots.

10. Ans. (A,B)

$\lambda^2 + 2\lambda (k + 1) + 4k = (8)(243)$

$\lambda^2 + 2\lambda k + 2\lambda + 4k = (8)(243)$

$\lambda(\lambda + 2k) + 2(\lambda + 2k) = (8)(243)$

$(\lambda + 2) + (\lambda + 2k) = (8)(243)$

$8(a + b + c)\, 8(ab + bc + ca) = (8)(243)$

$(a + b + c)(ab + bc + ca) = \frac{27}{8}.9$

$(a + b + c)(ab + bc + ca) = 9abc$

$a(b - c)^2 + b(c - a)^2 + c(a - b)^2 = 0$

$\Rightarrow a = b = c = \frac{3}{2}$

$R = \frac{a}{2\sin A} = \frac{\sqrt{3}}{2}$

$2R\cos B \cos C = \frac{\sqrt{3}}{4}$

11. Ans. (A,B,C,D)

$x = (\sqrt{3}+1)^{2n}$

given $a - 1 < x < a \Rightarrow a = \lfloor x \rfloor + 1$

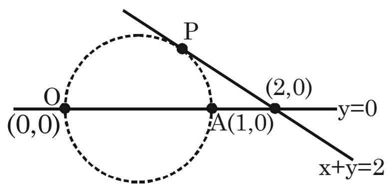
$x = \left((\sqrt{3}+1)^2\right)^n = 2^n (2+\sqrt{3})^n$

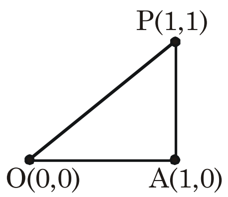
using pseudo function we get

$\lfloor x \rfloor + 1 = 2.\left[2^n\left({}^nC_0 2^n + {}^nC_2 2^{n-2}.3^1 + ... + ...\right)\right]$

$\therefore$ all are correct.

12. Ans. (A,C)





$\angle OPA$ is maximum $\therefore$ circle passing through

O,A,D touches x + y = 2 at P

use : $OB.OA = (PB)^2$

$\Rightarrow PB = \sqrt{2}$

$\Rightarrow$ (1,1) or (3,–1)

for maximum angle we take P(1,1)

$\alpha = 1 = \beta$

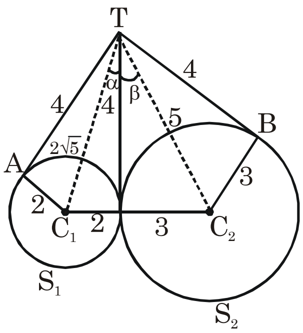
$\angle OPA = \frac{\pi}{4} = \tan^{-1}\frac{1}{1}$

a = b = 1

13. Ans. (A,B,C,D)

All are incorrect statements

14. Ans. (B,D)

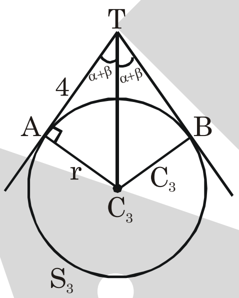


in ΔTC_1C_2

$\cos(\alpha+\beta) = \frac{20+25-25}{2.2\sqrt{5}.5}$

$\cos(\alpha+\beta) = \frac{1}{\sqrt{5}}$

$\tan(a+\beta) = 2$



$\Delta TAC_3 : \tan(\alpha+\beta) = \frac{r}{4}$

r = 8

Circumcircle of ΔTAB passes through centre C_3 and TC_3 is diameter

$\therefore$ radius $r' = \frac{TC_3}{2} = \frac{\sqrt{16+r^2}}{2} = \sqrt{20}$

Area of circumcircle of $\Delta TAB = \pi(r')^2 = 20\pi$

Paragraph for Question 15 to 16

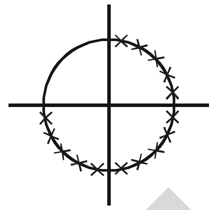
Locus of A : $\sin^{-1}x + \cos^{-1}0 = \sin^{-1}y$

$\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}y = 0$

$\sin^{-1}x + \cos^{-1}y = 0 \Rightarrow x^2 + y^2 = 1$

it is possible when $x \in [-1,0]$ and $y \in [0,1]$

Point A moves on circle $x^2 + y^2 = 1$ in II[nd] quadrant $L = \frac{\pi}{2}$



Locus of B : $\sin^{-1}Px + \cos^{-1}Pxy = \sin^{-1}y$

$\cos^{-1}Pxy = \sin^{-1}y - \sin^{-1}Px$

taking cos in both sides

$\cos(\cos^{-1}Pxy) = \cos(\sin^{-1}y)\cos(\sin^{-1}Px) + \sin(\sin^{-1}y)\sin(\sin^{-1}Px)$

$Pxy = \sqrt{1-y^2}\sqrt{1-P^2x^2} + Pxy$

$\because y^2 = 1$ or $P^2x^2 = 1$

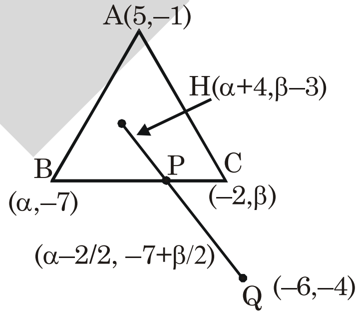
for only two straight lines

$P^2 = 0 \Rightarrow P = \{0\}$

15. Ans. (D)

16. Ans. (C)

Paragraph for Question 17 to 18



Let P is midpoint of

use : $m_{BH}.m_{AC} = -1 \Rightarrow \beta^2 + 5\beta - 24 = 0$

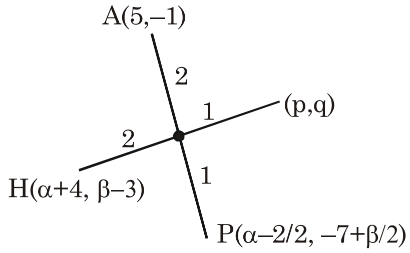
$\Rightarrow \beta = 3$ or -8

$m_{CH}.m_{AB} = -1 \Rightarrow \alpha^2 + \alpha - 12 = 0$

$\Rightarrow \alpha = 3$ or -4

$\beta^2 - \alpha^2 + 5\beta - \alpha = (\beta^2 + 5\beta) - (\alpha^2 + \alpha)$

$= 24 - 12 = 12$



$\therefore \frac{\alpha+3}{3} = \frac{2P+\alpha+4}{3} \Rightarrow P = -\frac{1}{2}$ and

$\frac{\beta-8}{3} = \frac{2q+\beta-3}{3} \Rightarrow q = -\frac{5}{2}$

17. Ans. (A)

18. Ans. (D)