LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced)

Test Type : **ALL INDIA OPEN TEST** (MAJOR) Test Pattern : JEE-Advanced

PAPER-1

PART-1 : MATHEMATICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,D	A,B,C,D	A,B,C	A	A,C,D	C,D	A,C,D	C	A	C
	Q.	11	12	13							
	A.	B	D	C							
SECTION-IV	Q.	1	2	3	4	5					
	A.	8	5	2	7	6					

SOLUTION

SECTION-I

1. Ans. (A,B,D)

Sol. P(Ram gets six on third throw) $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$.

P(shyam gets six on or after third throw)

$= \frac{25}{211}$

Let 'P' be the probability for event in option (D).

$$P = \underbrace{\frac{1}{6}\times\frac{1}{6}}_{\text{both obtain six}} + \underbrace{\left(\frac{1}{6}\times\frac{5}{6}\right)}_{\text{atleast one of them obtain six}}.2 + \underbrace{\frac{5}{6}.\frac{5}{6}}_{\text{None obtain six}} \times P \Rightarrow P = \frac{8}{33}$$

2. Ans. (A,B,C,D)

Sol. Consider the interval [0, 1]

$$g'(c_1) = \frac{g(x)}{x}$$

(By LMVT) where $x \in (0, 1]$

$|g(x)| = |xg'(c_1)|$

$|g(x)| \le |x||g(c_1)| = |x||c_1||g(c_2)|$

{Again using LMVT}

$0 \le |g(x)| \le |x||c_1||c_2| - |c_n||g(c_n + 1)|$

where $0 < c_n < c_{n-1} ..... < c_2 < c_1 < x < 1$

$\because$ 'n' can be increased indefinitely which implies $c_n \to 0$ & as g(x) is continuous

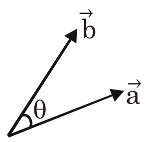
$|g(x)| = 0 \ \forall \ x \in [0, 1]$

same argument can be applied in [1, 2] starting with $|g(x)| \le |x - 1||g(c_1)|$

$\Rightarrow g(x) = 0$ for all x.

3. Ans. (A,B,C)

Sol. $\cos\theta = \frac{11}{14}$



$\vec{a} = \hat{i} + x\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} + (4x - 2)\hat{j} + 2\hat{k}$

$2|\vec{a}| = |\vec{b}|$

$4(1 + x^2 + 9) = (16 + (4x - 2)^2 + 4)$

$3x(x - 2) + 2(x - 2) = 0$

$x = 2, \ x = -\frac{2}{3}$

$$\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{11}{14}$$

$$\frac{4+x(4x-2)+6}{7(10+x^2)}=\frac{11}{14}$$

$17x^2 - 14x - 40 = 0$

$x = 2$

4. Ans. (A)

Sol. $$\left(\frac{x^2+2x+2}{x+1}\right)^{2018} = a_{2018}x^{2018} + a_{2017}x^{2017} + ..$$

$$..+a_1x+a_0+\frac{b_1}{x+1}+\frac{b_2}{(x+1)^2}+....+\frac{b_{2018}}{(x+1)^{2018}}$$

$$\left(x+1+\frac{1}{x+1}\right)^{2018} = {}^{2018}C_0(x+1)^{2018} + {}^{2018}C_1(x+1)^{2016}$$

$$+{}^{2018}C_2(x+1)^{2014}+...+{}^{2018}C_{1008}(x+1)^2+{}^{2008}C_{1009}$$

$$+{}^{2018}C_{2018}\left(\frac{1}{x+1}\right)^{2018}$$

Put x = 1

$$\sum_{i=0}^{2018} a_i + \sum_{i=1}^{2018}\frac{b_i}{2^i} = \left(2+\frac{1}{2}\right)^{2018} = \left(\frac{5}{2}\right)^{2018} \rightarrow (A)$$

Put x = 0,

$$2^{2018} = \left({}^{2018}C_0 + {}^{2018}C_1 + ... + {}^{2018}C_{1008} + {}^{20018}C_{1009}\right)$$

$$+\underbrace{\left({}^{2008}C_{1010} + {}^{2018}C_{1011} + ... + {}^{2018}C_{2018}\right)}_{\sum_{i=1}^{2018} b_i}$$

$$\Sigma b_i = \frac{2^{2018} - {}^{2018}C_{1009}}{2}$$

$$a_0 = \frac{2^{2018} + {}^{2018}C_{1009}}{2}$$

5. Ans. (A,C,D)

Sol. 2,3,4 ...... n–1 n

1,3,4..... n

....

1,2,3 n–1

In each sequence missing number can be accomodated in (n – 1) ways. But in every

⇒ consecutive pair one sequence is recounted

Hence

$f(n) = n(n-1) - (n-1) + 1,$

$f(n) = n^2 - 2n + 2$

6. Ans. (C,D)

Sol. $y = \sqrt{4x^2-4x+2} + |x|$

$= \sqrt{(2x-1)^2+1} + |x|$

if x < 0

$y = \sqrt{(2x-1)^2+1} - x$

$$y' = \frac{(2x-1).2}{\sqrt{(2x-1)^2+1}} - 1$$

$y' < 0 \ \forall\ x < 0$

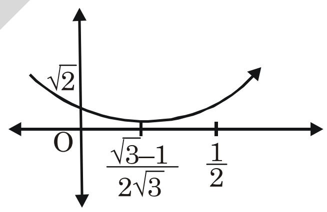
if x > 0

$y = \sqrt{(2x-1)^2+1} + x$

$$y' = \frac{2(2x-1)}{\sqrt{(2x-1)^2+1}} + 1$$

$$\Rightarrow y' > 0 \Rightarrow \frac{\sqrt{3}-1}{2\sqrt{3}} < x < \infty$$

$$y' < 0 \Rightarrow 0 < x < \frac{\sqrt{3}-1}{2\sqrt{3}}$$



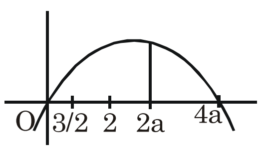
minima at $x = \dfrac{\sqrt{3}-1}{2\sqrt{3}}$

7. Ans. (A,C,D)

Sol. $f(x) = \log_a(4ax - x^2)$ $\left[\frac{3}{2}, 2\right]$

$a > 1 \Rightarrow f(x) = 4ax - x^2$

must be ↑ $\forall\ x \in \left[\frac{3}{2}, 2\right]$



$\Rightarrow 2a \geq 2 \Rightarrow a \geq 1$

$\Rightarrow a > 1$

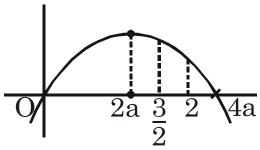
$0 < a < 1$

$\Rightarrow f(x) = 4ax - x^2$

must be $\downarrow \forall\ x \in \left[\frac{3}{2}, 2\right]$

$2a \le \frac{3}{2}$ & $4a > 2$

$a \le \frac{3}{4}$ & $a > \frac{1}{2}$

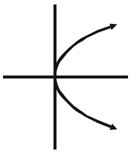


$\frac{1}{2} < a \le \frac{3}{4}$

$\left(\frac{1}{2}, \frac{3}{4}\right] \cup (1, \infty)$

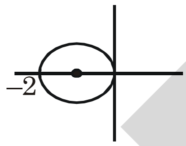
Answer Q.8, Q.9 and Q.10

(1) $y^2 = 4x$



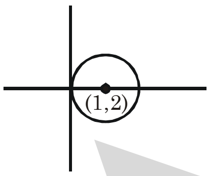
(i) $(x+1)^2 + 2y^2 = 1$

$$\frac{(x+1)^2}{1} + \frac{y^2}{1/2} = 1$$

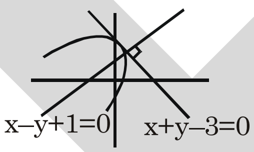


(2) $x^2 + y^2 - 2x = 0$

$(x - 1)^2 + y^2 = 1$

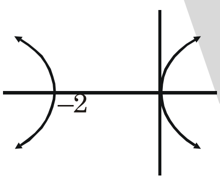


(ii) $(x + y - 3)^2 = -4\sqrt{2}(x - y + 1)$

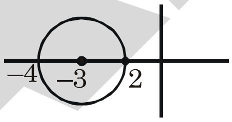


(3) $x^2 - y^2 + 2x = 0$

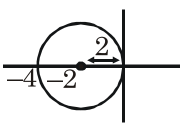
$(x + 1)^2 - y^2 = 1$



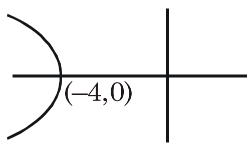
(iii) $(x + 3)^2 + y^2 = 1$



(4) $(x + 2)^2 + y^2 = 4$



(iv) $y^2 = -4(x + 4)$



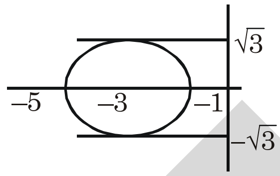
8. Ans. (C)

9. Ans. (A)

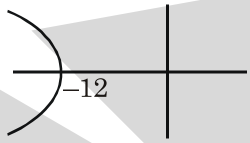
10. Ans. (C)

Answer Q.11,Q.12 and Q.13

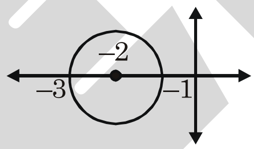
(1) $|z + 2| + |z + 4| = 4$



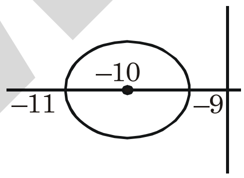
(2) $|z+13| = -11 - \text{Re}z$



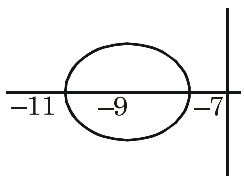
(3) $|z + 2| = 1$



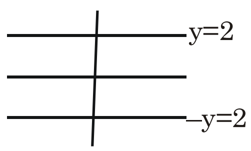
(4) $|z + 10| = 1$



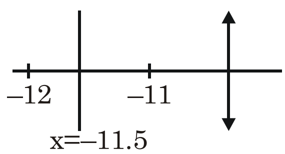
(i) $|z + 10| + |z + 8| = 4$



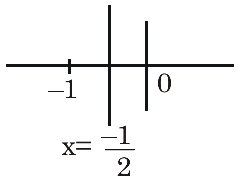
(ii) $z^2 + 8z + \bar{z}^2 + 8\bar{z} = 2|z|^2$



(iii) $|z + 12| = |z + 11|$



(iv) $|z| = |z+1|$



11. Ans. (B)

12. Ans. (D)

13. Ans. (C)

SECTION-IV

1. Ans. 8

Sol. $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

$$\alpha = \frac{\vec{r}.(2i+k)}{\sqrt{5}} = \frac{2a+b}{\sqrt{5}}$$

$$\beta = \frac{-2a+b}{\sqrt{5}}, \; r = c$$

$\therefore\ |a| = |\beta| = |\gamma|$

$$\Rightarrow \underbrace{|2a+b| = |-2a+b|}_{A} = \underbrace{|c\sqrt{5}|}_{B}$$

$$\left.\begin{aligned} a &= \pm\frac{\sqrt{5}}{3} \\ b &= 0 \\ c &= \pm\frac{2}{3} \end{aligned}\right\} \rightarrow 4$$

$$\left.\begin{aligned} c &= \pm\sqrt{\frac{1}{6}} \\ b &= \pm\sqrt{\frac{5}{6}} \\ a &= 0 \end{aligned}\right\} \rightarrow 4$$

$a = 0$

8 possible triplets.

2. Ans. 5

Sol. $$f(\sin\pi x) = \begin{cases} 0, & \sin\pi x < \frac{1}{2} \\ 1, & \sin\pi x \ge \frac{1}{2} \end{cases} = \begin{cases} 0, & 0 < x < \frac{1}{6}, \frac{5}{6} < x < 1 \\ 1, & \frac{1}{6} \le x \le \frac{5}{6} \end{cases}$$

$$\int_0^1 f(\sin\pi x)\,dx = \int_0^{1/6} 0 + \int_{1/6}^{5/6} 1 + \int_{5/6}^{1} 0 = \frac{5}{6} - \frac{1}{6} = \frac{2}{3}$$

$p = 2,\ q = 3 \Rightarrow p + q = 5$

3. Ans. 2

Sol. $$\lim_{x\to 0} \frac{\int_0^{\log(1+x)} (1-\tan 2y)^{1/y}}{\sin x}$$

$$L = \lim_{x\to 0} \left(1 - \tan(2\log(1+x))\right)^{\frac{1}{\log(1+x)}}$$

(Using L-Hospital)

$= e^{L'}$

$$L' = \lim_{x\to 0} \frac{-\tan(2\log(1+x))}{2\log(1+x)} \times 2 = -2$$

$$\therefore\ L = \frac{1}{e^2}$$

4. Ans. 7

Sol. $A^2 = I,\ \lambda = 4$

$\therefore\ B^2 = 81I$

$$\therefore\ Tr\left(\frac{A^2 + B^2}{82}\right) + \lambda = 7$$

5. Ans. 6

Sol. $E = |z+1| + |z|\left|z + \frac{1}{z} - 1\right|$

$E = |(z+1)| + |2Re(z) - 1| \qquad \because |z| = 1$

$= \sqrt{(x+1)^2 + y^2} + |2x-1|$

$E = \sqrt{2+2x} + |2x-1| \qquad$ let $2x - 1 = t$

$E = \sqrt{t+3} + |t| \qquad t \in [-3,1]$

Range of E is $\left[\sqrt{3}, \frac{13}{4}\right]$

PART-2 : PHYSICS — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,B,C	B,C	A,B,C,D	A,C	A,C	A,C	A,B,D	B	B	A
	Q.	11	12	13							
	A.	C	A	B							
SECTION-IV	Q.	1	2	3	4	5					
	A.	8	2	3	3	3					

SOLUTION

SECTION-I

1. Ans. (A,B,C)

Sol. $p = \frac{2Ze}{5}(b^2 - a^2)$ ....(i)

$$p = \frac{2Ze}{5}\left(\frac{(R_0 + \delta R_0)^3 - R_0^3}{b}\right)$$

$$p = \frac{2Ze}{5}\left(\frac{3R_0\delta R_0(R_0 + \delta R_0}{b}\right)$$

$$p = \frac{6Ze}{5}R_0^2\left(\frac{\delta R_0}{R_0}\right) \quad ....(ii)$$

By camparing (i) and (ii)

$$b^2 - a^2 \cong 3R_0^2\left(\frac{\delta R_0}{R_0}\right)$$

$(R_0 + \delta R_0)^2 - a^2 = 3R_0\delta R_0$

$a^2 = R_0^{\ 2} + \delta R_0^{\ 2} + 2R_0\delta R_0 - 3R_0\delta R_0$

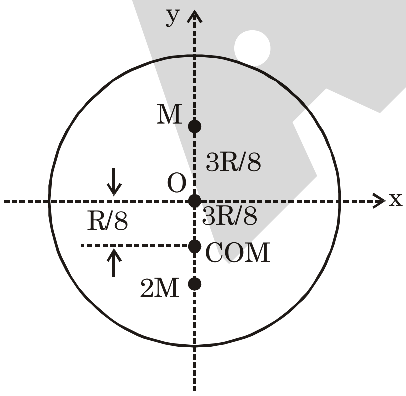
$= R_0^{\ 2} - R_0\delta R_0 \quad \{\delta R_0^{\ 2} \to 0\}$

$$= R_0^2\left(1 - \frac{\delta R_0}{R_0}\right)$$

2. Ans. (B,C)

3. Ans. (A,B,C,D)

Sol.



Where $M = f\left(\frac{2}{3}\pi R^3\right)$

$$y_{cM} = \frac{M(3R/8) + 2M(-3R/8)}{3M}$$

$$y_{cM} = -\frac{R}{8}$$

For equillibrium, F_B = weight of ball

$$\rho\left(\frac{4}{3}\pi R^3\right) = \rho\left(\frac{2}{3}\pi R^3\right) + 2\rho\left(\frac{2}{3}\pi R^3\right)$$

$4\rho_0 = 6\rho$

$$\rho = \frac{2\rho_0}{3}$$

$$I_0 = \frac{2M}{5}R^2 + \frac{2(2M)}{5}R^2 \quad \text{where } M = \frac{2}{3}\pi R^3\rho$$

$$I_0 = \frac{6MR^2}{5}$$

$I_{cm} = I_0 - md_2$

$$= \frac{6MR^2}{5} - 3M\left(\frac{R}{8}\right)^2$$

$$= \frac{6MR^2}{5} - \frac{3MR^2}{64} = \frac{369}{320}MR^2$$

$$= \frac{369}{320}R^2\left(\frac{2}{3}\pi R^3\rho\right)$$

$$I_{cm} = \frac{123}{160}\rho\pi R^5$$

4. Ans. (A,C)

Sol. $y_1 = 0.02 \sin\left[400\pi\left(\frac{x}{330} - t\right)\right]$

$$y_2 = 0.02\sin\left[404\pi\left(\frac{x}{330} - t\right)\right]$$

$y_1 + y_2 = 2(0.02)$

$$\sin\left[404\pi\left(\frac{x}{330} - t\right)\right]\cos\left[2\pi\left(\frac{x}{330} - t\right)\right]$$

$$= 0.04\sin\left(\frac{402}{330}\pi x - 402\pi t\right)\cos\left(\frac{2\pi x}{330} - 2\pi t\right)$$

5. Ans. (A,C)

Sol. If x be the initial length of the gas chambers & A be the area of cross section of the cylinder then,

$$x = \frac{V_0}{A} \text{ Also } P_0A = 2kx$$

$$\frac{P_0}{x} = 2kx \Rightarrow P_0V_0 = 2kx^2$$

$$\therefore \frac{1}{2}k(2x)^2 = \frac{1}{2}K4x^2 = 2kx^2 = P_0V_0$$

Similarly, $P_fA = 2k\frac{v_f}{A}$

$$P_0A = 2k\frac{V_0}{A}$$

$$\Rightarrow \frac{P_f}{P_0} = \frac{V_f}{V_0} \Rightarrow P = \frac{P_0}{V_0}v$$

Now $\therefore$ Q = dU + W

$$= 2\frac{5}{2}(P_1V_1 - P_0V_0) + w$$

$= 12\ P_0V_0$

6. Ans. (A,C)

7. Ans. (A,B,D)

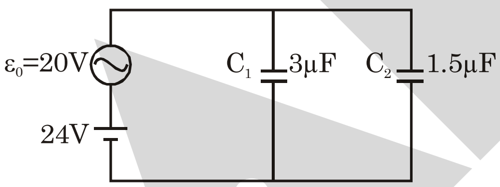
Sol. E = 24 + 20 cos120πt

Charge on C_2

$Q_2 = EC_2$

= (24 + 20cos120πt)1.5μC

= 36 + 30cos120πtμC



$$i_2 = \frac{dQ_2}{dt} = -30 \times 120\pi\sin(120\pi t)$$

$$i_1 = \frac{dQ_1}{dt} = \frac{d}{dt}(24 + 20\cos(120\pi t)3\mu C$$

= –60 × 120πtsin(120πt)

$i = i_1 + i_2$ = –90 × 120πtsin(120πt)

i = 33.9 sin(120πt + π) mA

Minimum energy stored

$$= \frac{1}{2}CV^2 = \frac{1}{2}(3 + 1.5)(4)^2\mu J$$

= 36μJ

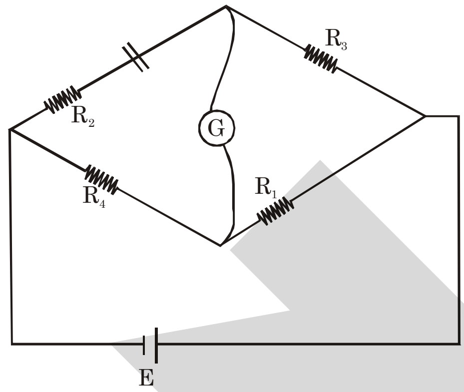
8. Ans. (B)

9. Ans. (B)

10. Ans. (A)

11. Ans. (C)

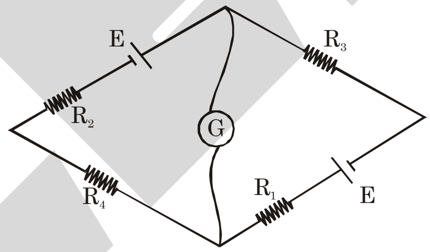
Sol. S_1 closed & S_2 & S_3 open



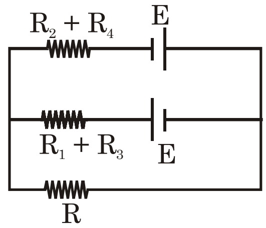
$$\frac{R_2}{R_4} = \frac{R_3}{R_1} \text{ (wheat stone bridge)}$$

12. Ans. (A)

Sol. (S_1 open, S_2, S_3 = Closed)



⇒



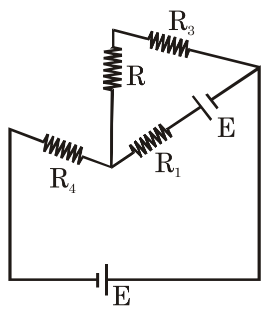
for current in galvanometer is zero.

$$E \Rightarrow 0 \Rightarrow \frac{E}{(R_2 + R_4)} - \frac{E}{(R_1 + R_3)} = 0$$

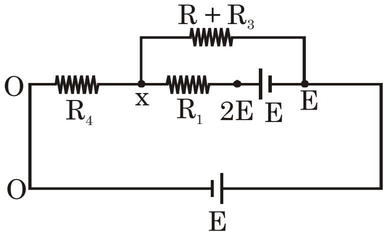
$R_2 + R_4 = R_1 + R_3$

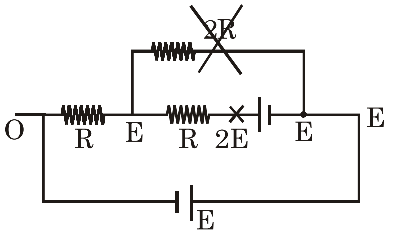
13. Ans. (B)

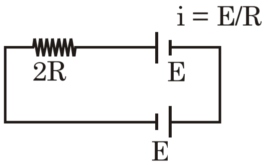
Sol. S_3 open, S_1 & S_2 closed



No potential doop across this net charge on capacitor







$$\frac{x-2E}{R_1}+\frac{x-E}{R+R_3}+\frac{x-0}{R_4}=0$$

$$\frac{x-2E}{R}+\frac{x-E}{2R}+\frac{x}{R}=0$$

$2x - 4E + 2x - 2E + 2x = 0$

$6x = 6E$

$x = E$

$R_3 = R_4 = R_1 = R$

$R_2 = 2R$

SECTION-IV

1. Ans. 8

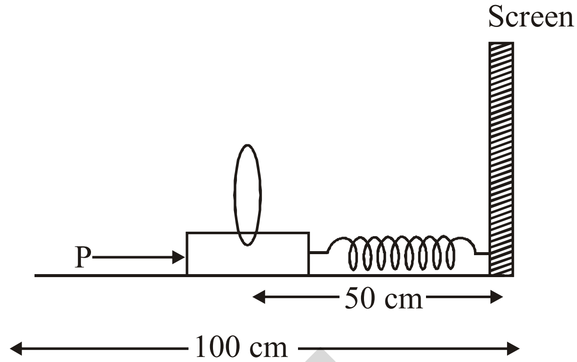
Sol. Let the distance of the lens from the object be l

when a real image is formed on the screen. Then

$$\frac{1}{100-l}-\frac{1}{-l}=\frac{1}{23}$$

On solving, we get $l = (50 \pm 10\sqrt{2})$ cm.

Now, if the lens performs SHM and a real image is formed after a fixed time gap, then this time gap must be one-fourth of the time period.



$\therefore$ Phase difference between the two positions of real image must be $\frac{\pi}{2}$. As the two positions are symmetrically located about the origin, phase difference of any of these positions from origin must be $\frac{\pi}{4}$.

$$\Rightarrow 10\sqrt{2}\text{ cm} = A\sin\frac{\pi}{4} \Rightarrow A = 20\text{ cm}$$

To achieve this velocity at the mean position,

$$\upsilon_0 = A\omega = A\sqrt{\frac{K}{m}}$$

$\therefore$ Required impulse $p = m\upsilon_0 = A\sqrt{K/m} =$ 8 kg m/s.

2. Ans. 2

Sol. The hand is a point mass which has a moment of inertia with respect to the center of the wheel mr^2 at the time of each impact. The angular momentum transferred from the hand to the wheel in the n-th hit (after the hit, the hand is at rest with respect to the point of impact) in mr $(w - v_n)$ where v_n is the velocity of the point of impact after the n-th hit. The angular momentum of the wheel after the n-th hit is therefore

$$L_n = mr\,(w - v_n) + L_{n+1}$$

That give a recurrent formula for the velocities of the point of impact based on the formula $L_n = I\omega_n$

$$Av_n = w - v_n + Av_{n-1}$$

where we used the substitution $A = I/(mr^2)$

Now, we will try to compute the first few terms of the progression. We get

$$v_1 = \frac{\omega}{1+A}, \qquad v_2 = \frac{\omega}{1+A}\left(1+\frac{A}{1+A}\right),$$

$$v_3 = \frac{\omega}{1+A}\left(1+\frac{A}{1+A}+\left(\frac{A}{1+A}\right)^2\right), \ldots$$

The formula for v_n will clearly be

$$v_n = \frac{\omega}{1+A}\left(1+\frac{A}{1+A}+\ldots..+\left(\frac{A}{1+A}\right)^{n-1}\right)$$

Summing up a geometric series and simplifying, we obtain

$$v_n - \omega\left(1-\left(\frac{A}{1+A}\right)^n\right).$$

The velocity is $v_{10} = 4.54\ m\cdot s^{-1}$.

3. Ans. 3

Sol. At depth x, the density is determined by

$$\rho(x) = \rho_1 + \frac{x}{h}(\rho_b - \rho_1)$$

For the rod to be at equilibrium, it is required that the total torque acting on the rod is zero, i.e.

$$\int dM = 0$$

We can express the elementary torque acting on an infinitely small section of the rod as $dM = x(dF_{vz} - dF_g)$ where the elementary forces dF_{vz} are given by

$$dF_{vz} = \frac{mg\rho(x)}{\rho_r h}dx,$$

$$dF_g = \frac{mg}{h}dx$$

where m is the mass of the rod. Integrating from 0 to hcosφ (where our x-axis is directed vertically downwards and hcosφ is the x-coordinate of the lower end of the rod), we have

$$\int_0^{h\cos\varphi} x\left(\frac{mg}{\rho_r \ell}\left(\rho_1 + \frac{x}{h}(\rho_b - \rho_1)\right) - \frac{mg}{\ell}\right)dx = 0,$$

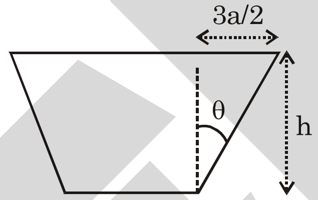
$$\frac{\rho_1}{2\rho_r \ell} + \frac{h\cos\varphi}{3\rho_r h\ell}(\rho_b - \rho_1) - \frac{1}{2\ell} = 0$$

It remains to express ρ_b and substitute the numerical values. Eventually, we obtain

$$\rho_b = \frac{3}{2\cos\varphi}(\rho_r - \rho_1) + \rho_1 = 999 kg\cdot m^{-3}$$

4. Ans. 3

Sol.



$$\tan\theta = \frac{3a}{2h}$$

$$2 \times 4a\ T \times \cos\theta = 4a\lambda g$$

$$\cos\theta = \frac{\lambda g}{2T}$$

$$\Rightarrow h = \frac{3\lambda ga}{2\sqrt{4T^2 - \lambda^2 g^2}}$$

5. Ans. 3

Sol. $y = \frac{D}{d}\left(\frac{1}{f}\sqrt{\frac{rRT}{M}}\right)$

$$y \propto \sqrt{T}$$

$$\frac{\Delta y}{y} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta y}{y}\times 100 = \frac{1}{2}\times 1$$

% charge in y = $\frac{1}{2}$

PART-3 : CHEMISTRY

ANSWER KEY

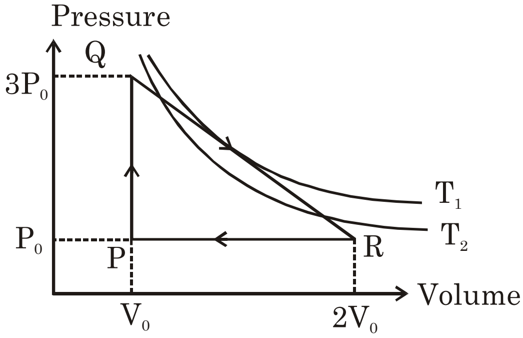
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,C,D	A,B,D	A,B,C	A	B,C,D	B,D	C	C	B
	Q.	11	12	13							
	A.	A	C	B							
SECTION-IV	Q.	1	2	3	4	5					
	A.	6	2	1	3	5					

SOLUTION

SECTION I

1. Ans.(A, C)

Sol.



(A) Consider two isotherms at T_1 & T_2 drawn like above. now $T_2 < T_1$. While moving from Q to R, we first encounter T_2 & then T_1, $\therefore$ A is correct.

(C) $|q_{PR}| = \Delta H = nCp\,(T_P - T_R)$

$$= (1)\left(\frac{5}{2}R\right)\left(\frac{P_0V_0}{(1)R} - \frac{2P_0V_0}{(1)R}\right)$$

$$= -\frac{5}{2}P_0V_0$$

$\therefore$ C is correct

(D) $\because$ cyclic process ; $\Delta U = 0$

$$q_{cycle} = q_{PQ} + q_{QR} + q_{RP} = -W_{cycle}$$

$$= -\frac{1}{2}(2P_0.V_0)$$

$$(1)\left(\frac{3}{2}R\right)\left(\frac{3P_0V_0}{R} - \frac{P_0V_0}{R}\right) + q_{QR} = -P_0V_0 + \left(\frac{5}{2}P_0V_0\right)$$

$$\Rightarrow 3P_0V_0 - \frac{5}{2}P_0V_0 + q_{QR} = P_0V_0$$

$$\Rightarrow q_{QR} = -\frac{3}{2}P_0V_0$$

2. Ans.(A, C, D)

Sol. Total charge passed $= \sum i.dt.$

$$= \frac{1}{2}\left(\frac{100}{1000}\times 10\right) + \left(\frac{100}{1000}\times 10\right)$$

$$= +\frac{1}{2}\left(\frac{100}{1000}\times 10\right) \text{ Amp. sec.}$$

= 2 Amp. sec. = 2 coulomb

(A) w = z.Q , z = electrochemical equivalent

$$= z = \frac{200\,\text{gms}}{2\,\text{coulomb}} = 100\,\text{gm/coulumb}$$

(C) Total charge needed = 2 coulumb

$$= \left(\frac{66.66}{1000} \times 30\right)\text{coulumb} = 2 \text{ coulumb.}$$

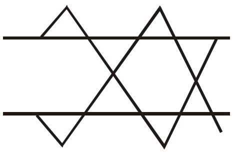
(D) w = (100 gm / coulumb) $\times \left[\left(\frac{1}{2} \times \frac{100}{1000} \times 10\right) + \left(\frac{100}{1000} \times 5\right)\right]$coulumb = 100 gm

3. Ans.(A, B, D)

Sol.

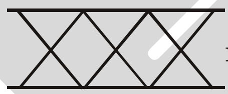
4. Ans.(A, B, C)

Sol. (A)



n = 5/2 = 2.5

(B)



n = 3

n = number of oxygen shared by pertetrahedra

5. Ans.(A)

Sol.

$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$

$\underline{sp^3d^2}$

$d_{x^2-y^2}, d_{z^2}$

$XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

$\underline{sp^3d}$

d_{z^2}

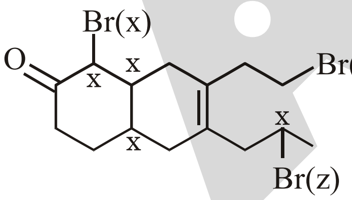
$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

$\underline{sp^3}$

None

6. Ans. (B, C, D)

Sol.



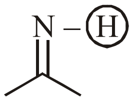
$Br_{(x)}$ most reactive for S_N2 due to –I eff. of $-\underset{\underset{O}{\|}}{C}-GP$

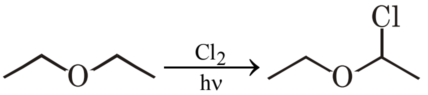
$Br_{(z)}$ most reactive for S_N1 due to carbocation stablised by 5α–H

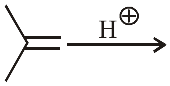
Total (4) chiral centre

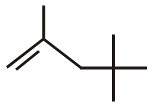
so total stereo isomer = 2^4 = 16

7. Ans. (B, D)

Sol. (A) 2 > 1  due to acidic hydrogen

(B)  α–halogenation

(C)  Dimerised major product



(D) Only glucose give asazone

8. Ans.(C)

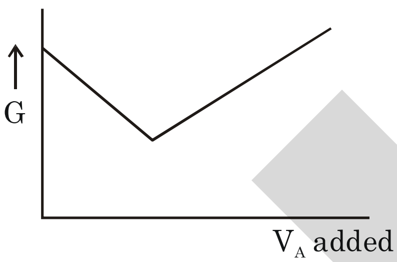
9. Ans.(C)

10. Ans.(B)

Sol.

	NaOH	+	HCl	→	NaCl	+	H_2O
t = 0	10 mmole		10 mmole				
t = 0	0		0		10 mole		[NaCl] = 0.01M

(neither ion hydrolysis to an appreciable extent)



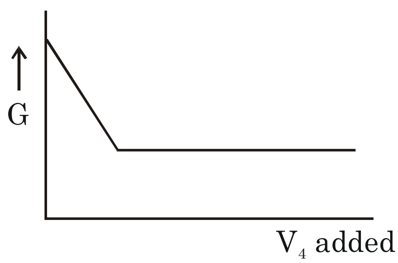
(I) , (i) , (b)

	NH_4OH	+	HCl	→	NH_4Cl	+	H_2O
t = 0	10 mmol		10 mmole				
t = 0	0		0		10 mmole		

$\Rightarrow$ $[NH_4Cl] = \dfrac{10}{1000} = 0.01M$

$\Rightarrow$ $pH = \dfrac{pk_w - pk_b - \log C}{2}$

$= \dfrac{14 - 5 - \log(0.01)}{2}$

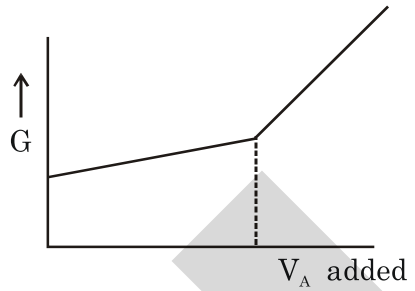


5.5 (cation hydrolysis)

(II), (iv), (c)

	$NaOH$ +	$CH_3COOH \rightarrow$	CH_3COONa +	H_2O
t= 0	10 mmole	10 m mole	0	0
t= 0	0	0	10 m mole	0

$$[CH_3COONa] = \frac{10\text{mmole}}{1000\text{ml}} = 0.01\text{M}$$



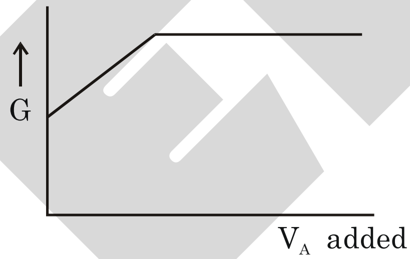
$$pH = \frac{pk_w - pk_b - \log C}{2} = \frac{14+5+\log(0.01)}{2}$$

= 8.5 (anion hydrolysis)

(II), (ii), (a)

	NH_4OH +	$CH_3COOH \rightarrow$	CH_3COON_4 +	H_2O
t= 0	10 mmole	10 m mole	0	0
t= 0	0	0	10 m mole	0

$$[CH_3COONa] = \frac{10\text{mmole}}{1000\text{ml}} = 0.01\text{M}$$



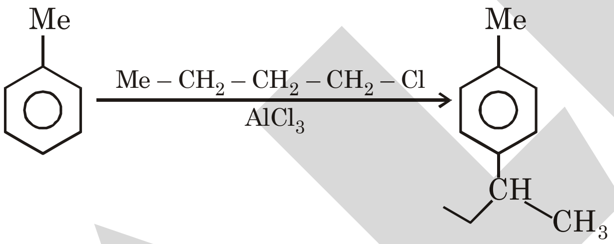
$$pH = \frac{pk_w + pk_A - pk_b}{2} = \frac{14+5-5}{2} = 7$$

= 8.5 (anion hydrolysis)

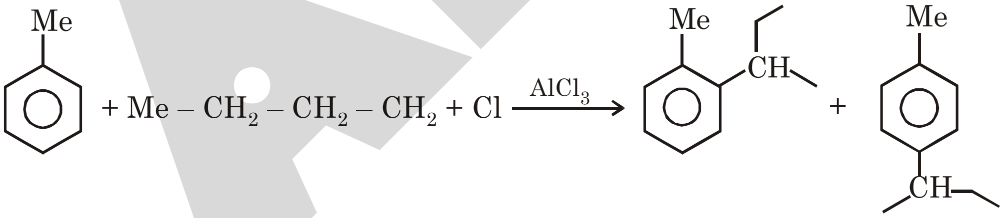
(IV), (iii), (c)

11. Ans. (A)

Sol.

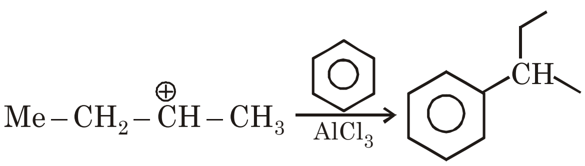


[I] – [D] – [P]



$$Me - CH_2 - CH_2 - Cl + AlCl_3 \longrightarrow Me - CH_2 - CH_2 - \overset{\oplus}{C}H_2 \xrightarrow{H-\text{shift}} Me - CH_2 - \overset{\oplus}{C}H - CH_3$$

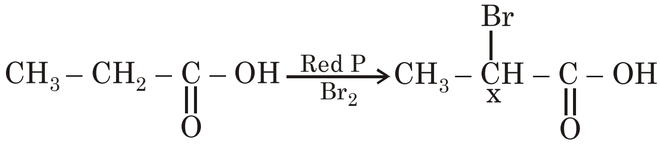
Carbocation intermediate



Electrophilic Substituting

I – D – P

12. Ans. (C)

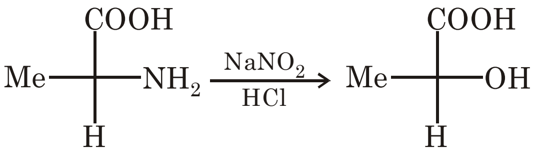
Sol. 

Optical active

Hell vholard zelenski not follows free radical mechanism

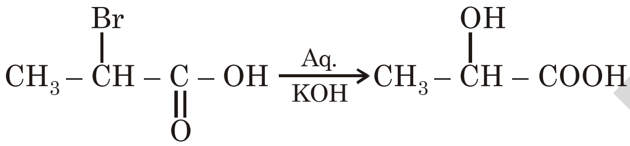
13. Ans. (B)

Sol. Product of IV is :



Diazotisation

product of III is



Optical active

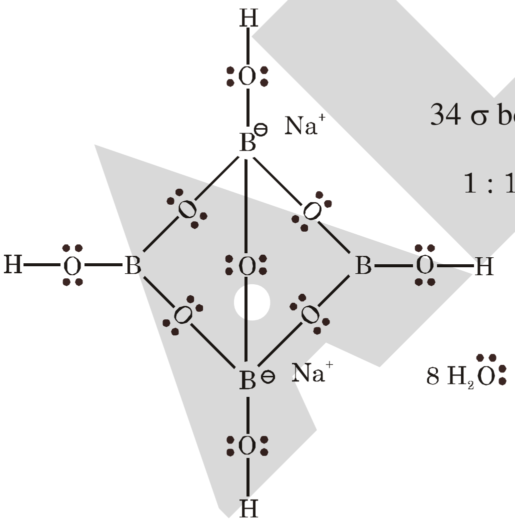
SECTION-IV

1. Ans. (6)

2. Ans.(2)

Sol. (b) and (d) have atleast three isomers

3. Ans.(1)

Sol. 

34 σ bond : 34 lone pairs

1 : 1 = 1

4. Ans.(3)

Sol. 1) $PCl_3 + 3H_2O \longrightarrow H_3PO_3 + 3HCl$

2) $PCl_5 + 4H_2O \longrightarrow H_3PO_4 + 5HCl$

3) $IF_7 + 4H_2O \longrightarrow HIO_4 + 7HF$

4) $IF_5 + 3H_2O \longrightarrow HIO_3 + 5HF$

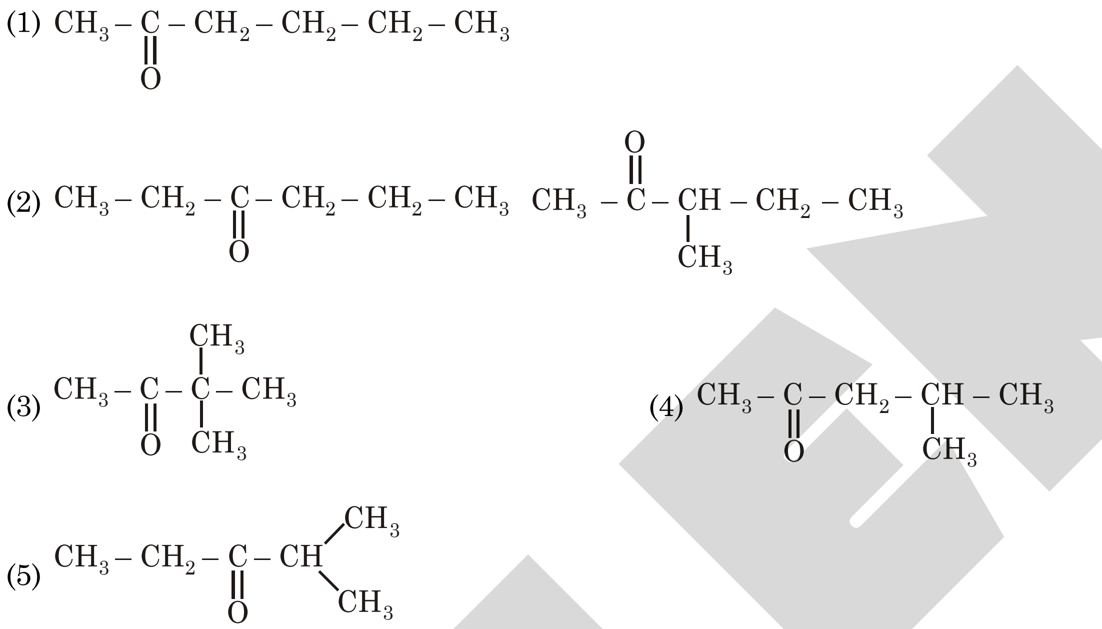
5) $P_4O_{10} + 6H_2O \longrightarrow 4H_3PO_4$

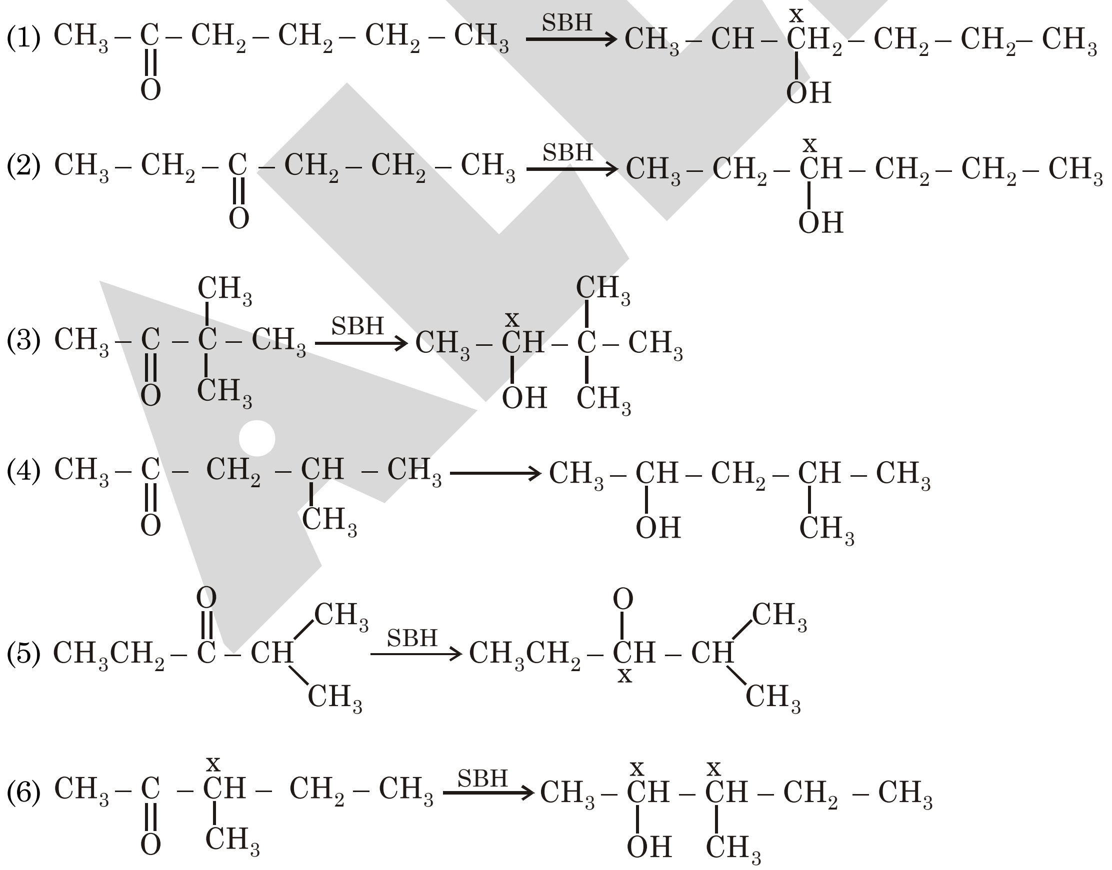
6) $H_2S_2O_7 + 2H_2O \longrightarrow 2H_2SO_4$

7) $H_4P_2O_8 + 2H_2O \longrightarrow 2H_3PO_4 + H_2O_2$

5. Ans. (5)

Sol. So ketone is $C_nH_{2n}O = 100$, n = 6





Not consider
due to 2–chiral

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced)

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Advanced

PAPER-2

PART-1 : MATHEMATICS **ANSWER KEY**

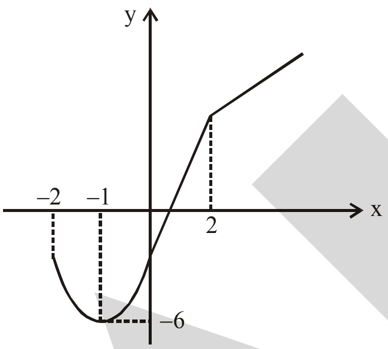
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	D	C	D	B	D	C	A,B,D	B,C	A,C
	Q.	11	12	13	14	15	16	17	18		
	A.	A,D	A,D	A,B,D	A,B,D	C	C	B	C		

SOLUTION

SECTION-I

1. Ans. (C)

Sol. $f(x)=\begin{cases}2x^2+4x-4 & x\in[-2,2]\\ 4x+4 & x\in(2,\infty)\end{cases}$



$\therefore$ Range : [–6, ∞)

2. Ans. (D)

Sol. Line $\frac{x-k}{1}=\frac{y-2}{k}=\frac{z-k}{2}$

Plane $2x - 4y + z = 7$

$\therefore\ 2k - 8 + k = 7 \Rightarrow k = 5$

& $2 - 4k + 2 = 0 \Rightarrow k = 1$

$\Rightarrow$ no value of k exist.

3. Ans. (C)

Sol. $h^2 + (k + 1)^2 + h^2 + (k - 2)^2 = 3[h^2 + (k - 1)^2]$

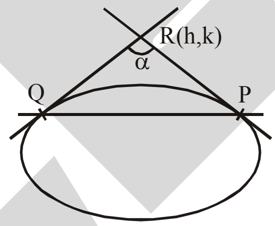
$2h^2 + 2k^2 - 2k + 5 = 3h^2 + 3k^2 + 3 - 6k$

$\Rightarrow h^2 + k^2 - 4k - 2 = 0$

$\Rightarrow x^2 + y^2 - 4y - 2 = 0$

4. Ans. (D)

Sol.



Chord of contact of R

$hx + 2ky = 6$

compare with $\frac{x}{2}\cos\theta + y\sin\theta = 1$

$\frac{2h}{\cos\theta}=\frac{2k}{\sin\theta}=6$

$h^2 + k^2 = 9$

which is director circle of $x^2 + 2y^2 = 6$.

5. Ans. (B)

Sol. $\lim_{a\to\infty}\frac{\int_0^{1/a^2}\frac{\cos^{-1}ax}{\sec^{-1}(ax+2)}dx}{\frac{1}{a^2}}$, let ax = t

$=\lim_{a\to\infty}\frac{\int_0^{1/a}\frac{\cos^{-1}t}{\sec^{-1}(t+2)}dt}{\frac{1}{a}}$

$=\lim_{a\to\infty}\frac{\cos^{-1}\frac{1}{a}}{\sec^{-1}\left(\frac{1}{a}+2\right)}=\frac{\frac{\pi}{2}}{\frac{\pi}{3}}=\frac{3}{2}$

6. Ans. (D)

$x^2 + y^2 = 5 \Rightarrow (x-y)^2 = 5 - 2xy$

$V = \pi x^2 y - \pi y x^2 = \pi xy\sqrt{(5-2xy)}$

Let xy = t

$V = \pi t\sqrt{(5-2t)}$

$\Rightarrow \frac{dv}{dt} = \pi\left[\sqrt{5-2t} + \frac{(-2)t}{2\sqrt{5-2t}}\right] = 5 - 2t - t = 0$

$\Rightarrow \quad t = \frac{5}{3} \quad \Rightarrow \quad xy = \frac{5}{3}$

7. Ans. (C)

Sol. Let a point on $y^3 = x^4$ be (t^3, t^4)

$3y^2y' = 4x^3$

$\Rightarrow \quad y' = \frac{4x^3}{3y^2} \quad \Rightarrow \quad y' = \frac{4}{3}t$

Equation of tangent is

$y - t^4 = \frac{4t}{3}(x - t^3)$

$\because$ it is a normal to $x^2 + y^2 - 2x = 0$

$\therefore$ it must pass through (1, 0)

$\Rightarrow \quad -\frac{3}{4}t^3 = 1 - t^3 \Rightarrow \quad \frac{t^3}{4} = 1$

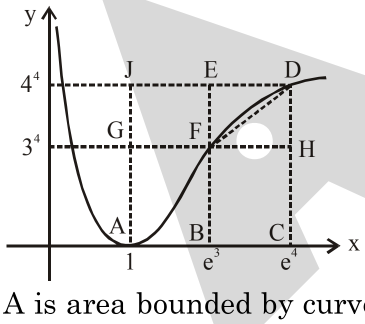
$\Rightarrow \quad t^3 = 4$

Now $m = \frac{4t}{3}$

$\Rightarrow \quad \left(\frac{3m}{4}\right)^3 = t^3 = 4$

8. Ans. (A,B,D)

Sol.



A is area bounded by curve between x = 1, $x = e^4$

$x = e^3$ is point of inflection

Now, A < Ar.(□ABGF) + Ar.((□BCDE)

$\Rightarrow \quad A < (e^3 - 1)3^4 + (e^4 - e^3)4^4$

$\Rightarrow \quad A < 256e^4 - 175e^3 - 81$ (Ans. A)

Also, A < Ar.(□ACDJ)

$\Rightarrow \quad A < (e^4 - 1)4^4$

$\Rightarrow \quad A < 256(e^4 - 1)$ (Ans. B)

Again, A > Ar.(trapezium BFDC)

$\Rightarrow \quad A > \frac{1}{2}(3^4 + 4^4)(e^4 - e^3)$

$\Rightarrow \quad A > \frac{337}{2}(e^3(e-1))$ (Ans. D)

9. Ans. (B,C)

Sol. Put z = x + iy, (x,y $\in$ R)

$\Rightarrow \quad x^2 - y^2 + 2ixy - 2iy + 2|y|i = 8i$

$\begin{cases} x^2 - y^2 = 0 \Rightarrow x = y \text{ or } x = -y \\ 2xy - 2y + 2|y| = 8 \end{cases}$

$\Rightarrow \quad xy - y + |y| = 4 \quad ....(1)$

Case-I : when x = y

$\Rightarrow \quad x^2 - x + |x| = 4$

(i) if $x \geq 0$, then x = 2, y = 2

$\Rightarrow \quad z = 2(1 + i)$

(ii) if $x < 0 \Rightarrow y < 0$ (rejected)

Case-II : when x = –y

$\Rightarrow \quad -x^2 + x + |x| = 4$

$x^2 - x - |x| = -4$

(i) if $x \geq 0$, then $x^2 - 2x + 4 = 0$

$\Rightarrow \quad x \in \phi$

(ii) if $x < 0 \Rightarrow x^2 = -4$

$\Rightarrow \quad x \in \phi$

10. Ans. (A,C)

Sol. $\frac{-y\,dy}{\sqrt{1-y^2}} = xdx$

$\frac{-2y}{2\sqrt{1-y^2}}dy = xdx$

$\left(\frac{-2y}{2\sqrt{1-y^2}}\right)dy = xdx$

$d\left(\sqrt{1-y^2}\right) = xdx$

$\sqrt{1-y^2} = \frac{x^2}{2} + C$

put x = 0, y = 1 $\Rightarrow$ C = 0

$\Rightarrow \quad 1 - y^2 = \frac{x^4}{4} \Rightarrow \frac{x^4}{4} + y^2 = 1$

11. Ans. (A,D)

Sol. $\tan^{-1}x + \tan^{-1}\frac{1}{y} = \tan^{-1}3$

$\frac{x + \frac{1}{y}}{1 - \frac{x}{y}} = 3$

$\Rightarrow \quad xy + 1 = 3y - 3x$

$\Rightarrow \quad x = \frac{3y-1}{y+3} = 3 - \frac{10}{y+3}$

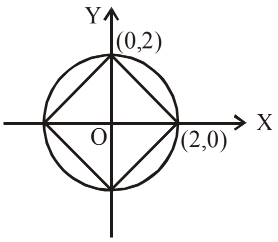
$y + 3 = 5, 10$

$y = 2, 7 \Rightarrow x = 1, 2$

12. Ans. (A,D)

Sol. Locus of P is a circle $X^2 + Y^2 = 4$

and locus of Q is $|X| + |Y| = 2$



13. Ans. (A,B,D)

Sol. $x^2 - 3x + b = 0 \begin{cases} \alpha \\ \beta \end{cases}$ or $\begin{matrix} \alpha \\ 3\beta \end{matrix}$

$x^2 - ax + 6 = 0 \begin{cases} \alpha \\ 3\beta \end{cases}$ or $\begin{matrix} \alpha \\ \beta \end{matrix}$

Case-I : $\begin{cases} \alpha + \beta = 3 \\ \alpha.3\beta = 6 \end{cases}$

$\Rightarrow \alpha = 1, \beta = 2$ or $\alpha = 2, \beta = 1$

$\Rightarrow a = 7, b = 2$ and $a = 5, b = 2$

Case-II : $\begin{cases} \alpha + 3\beta = 3 \\ \alpha\beta = 6 \end{cases}$

$\Rightarrow (3 - 3\beta)\beta = 6$

$\Rightarrow \beta^2 - \beta + 2 = 0$

$\Rightarrow$ no real value of β.

14. Ans. (A,B,D)

Sol. adj(M) = 2N, adj(N) = M

$\Rightarrow \quad |adjM| = 8|N|$ and $|adjN| = |M|$

$\Rightarrow \quad |M|^2 = 8|N|$ and $|N|^2 = |M|$

$\Rightarrow \quad |N| = 2$

Now, MN = adj(N).M = $|N|$I

$\Rightarrow \quad$ MN = 2I $\Rightarrow$ (B)

Now adj(M^2N) + adj(MN^2)

= adj(M.2I) + adj(2IM)

= 4(2N) + 4(M)

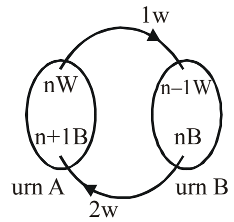
Now, adj(MN^{-1}) = adj($2N^{-2}$) $\Rightarrow$ (A)

= $4(adjN)^{-2} = 4M^{-2}$ $\Rightarrow$ (C)

Paragraph for Question 15 and 16

15. Ans. (C)

Sol. Case : I



$$\frac{{}^{n}C_1}{{}^{2n+1}C_1} \times \frac{{}^{n+2}C_2}{{}^{2n+2}C_2} + \frac{{}^{n+1}C_1}{{}^{2n+1}C_1} \times \frac{{}^{n+1}C_1 . {}^{n+1}C_1}{{}^{2n+2}C_2} = \frac{13}{25}$$

$\Rightarrow 29n^2 - 46n - 24 = 0$

$\Rightarrow (n-2)(29n+12) = 0$

$n = 2$

16. Ans. (C)

Sol. $$\frac{\frac{2}{5} \times \frac{6}{15}}{\frac{2}{5} \times \frac{6}{15} + \frac{3}{5} \times \frac{9}{15}} = \frac{4}{13}$$

Paragraph for Question 17 and 18

17. Ans. (B)

Sol. $2f(x).f'(x) - 2f'(x) - \frac{2}{2\sqrt{x}} = 0$

$2(f(x)-1)f'(x) = \frac{1}{2\sqrt{x}}$

integrating both the sides

$(f(x)-1)^2 - \sqrt{x} + C$

$\because f(0) = 2 \; c = 1$

$\Rightarrow (f(x)-1)^2 = \sqrt{x} + 1$

$\Rightarrow f(x) = 1 + \sqrt{\sqrt{x}+1}$

Now, $\lim\limits_{x \to 0^+} \frac{f(x)-2}{\sqrt{x}} = \lim\limits_{x \to 0^+} \frac{\sqrt{\sqrt{x}+1} - 1}{\sqrt{x}}$

$= \lim\limits_{x \to 0^+} \frac{\sqrt{x}+1-1}{\sqrt{x}\left(\sqrt{\sqrt{x}+1}+1\right)} = \frac{1}{2}$

18. Ans. (C)

Sol. $\because g'(y) = \frac{1}{f'(x)} \Rightarrow g'(4) = \frac{1}{f'(64)}$

$(\because f(64) = 4)$

$= \frac{1}{\frac{1}{96}} = 96$

PART-2 : PHYSICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	A	B	C	A	A	B,C	B,C,D	B,D
	Q.	11	12	13	14	15	16	17	18		
	A.	A,D	D	A,C	B,C,D	B	A	B	D		

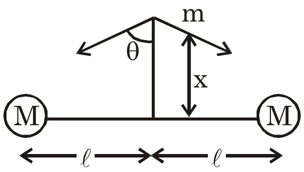
SOLUTION

SECTION-I

1. Ans. (C)

Sol. Force on m due to M(F) = $\frac{GMm}{(\ell^2+x^2)}$

Net downward force on m = 2Fcosθ



$= 2\frac{GMm}{\ell^2+x^2}\cdot\frac{x}{\sqrt{\ell^2+x^2}}$

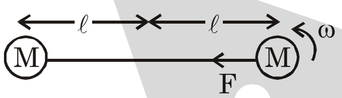
$= \frac{2GMmx}{(\ell^2+x^2)^{3/2}}$

if x << ℓ

$F_{net} = \frac{2GMmx}{\ell^3} = ma$

$a = \frac{2GMx}{\ell^3} \Rightarrow \boxed{T_P = 2\pi\sqrt{\frac{\ell^3}{2GM}}}$

For T_S



$F = \frac{GMM}{(2\ell)^2} = M\omega^2\ell$

$\Rightarrow \frac{GM}{4\ell^2} = \omega^2\ell \quad \Rightarrow \omega = \sqrt{\frac{GM}{4\ell^3}}$

$\Rightarrow T_S = 2\pi\sqrt{\frac{4\ell^3}{GM}}$

$\frac{T_P}{T_S} = \frac{1}{2\sqrt{2}}$

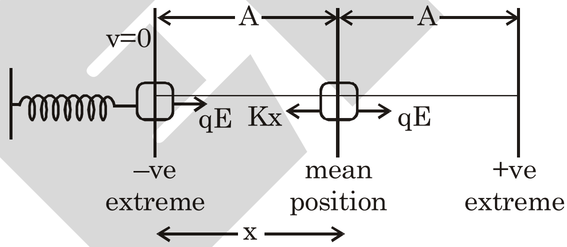
2. Ans. (C)

Sol. When identical resistor is inserted in BD then it forms a wheatstone bridge due to which current in the bulb becomes zero and it becomes dark.

3. Ans. (A)

Sol. For mean position : qE = kx

$x = \frac{qE}{k}$ = Amplitude A



$\Rightarrow \boxed{A = \frac{qE}{k}}$

Net force on block at any instant

$F_{net} = qE - kx$

$F_{net} = -k\left(x - \frac{qE}{k}\right)$

$= -k(x - x_0)$

$F_{net} = -kX$

$ma = -kX$

$a = -\frac{k}{m}.X$

$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

4. Ans. (B)

Sol. Let area of needle be A and length be ℓ.
For needle to remain in equilibrium
Force due to surface tension ≥ weight of needle

$2S\ell \geq \rho(A\ell)g$

$A \leq \frac{2S}{\rho g}$

$$\frac{\pi D^2}{4} \le \frac{2S}{\rho g} \quad \Rightarrow D^2 \le \frac{8S}{\pi \rho g}$$

on solving $\boxed{D \le 1.53\text{mm}}$ & independent of length.

5. Ans. (C)

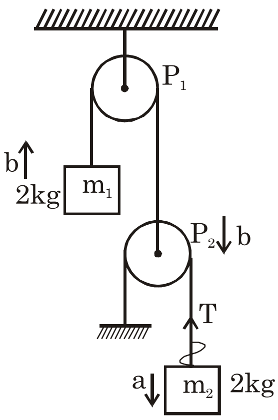
6. Ans. (A)

7. Ans. (A)

Sol. We have

$2T - 20 = 2b$ (i)

$20 - T = 2a$ (ii)



$a - 2b = 0 \quad \Rightarrow \; a = 2b$ (iii)

$2T - 20 = 2b$

$40 - 2T = 8b$

$20 = 10b \Rightarrow b = 2\text{m/s}^2$

$\therefore \; a = 4 \text{ m/s}^2$

Also T = 12 N

$$\therefore \; v_{wave} = \sqrt{\frac{12}{3\times10^{-2}}} = 20\text{m/s}$$

$a_{pulley} = 2 \text{ m/s}^2$

$\therefore \; 0.2 = 20t + \frac{1}{2}2t^2$

$\Rightarrow t^2 + 20t - 0.2 = 0$

t = 0.01 s

8. Ans. (B, C)

Sol. Since no heat transferred $\Rightarrow$ process is adiabatic

$\therefore \; W_{atm} + W_{gas} = \Delta KE_{bullet}$

$$W_{gas} = \frac{P_iV_i - P_fV_f}{\gamma - 1} = \frac{8P_0 \times 5\times10^{-4} - P_0V_f}{0.5}$$

Also $8P_0 \times (5 \times 10^{-4})^{\gamma} = P_0 V_f^{\gamma}$

$\Rightarrow \; V_f = 8^{2/3} \times 5\times10^{-4}\text{ m}^3 = 20\times10^{-4}\text{ m}^3$

$$\therefore \; W_{gas} = \frac{8P_0 \times 5\times10^{-4} - P_0 \times 20\times10^{-4}}{0.5}$$

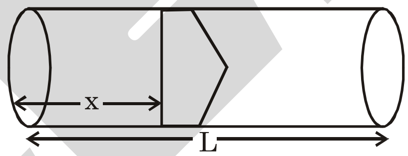
$$= \frac{P_0 \times 10^4 \times 20(2-1)}{0.5} = 400\text{J}$$

Given $10^{-2} \times x = 5 \times 10^{-4} \Rightarrow x = 5 \times 10^{-2}$ m

Also 10^{-2} L = $20 \times 10^{-4} \Rightarrow L = 20 \times 10^{-2}$ m

$\therefore \; W_{atm} =$

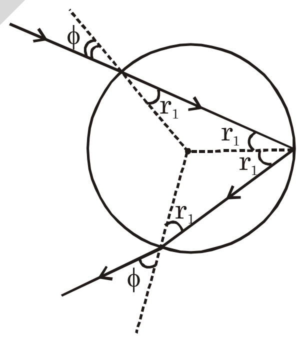
$P_0\left(V_f - V_i\right) = -P_0(15\times10^{-4}) = -150\text{ J}$



$\therefore \; \Delta KE_{bullet} = 250$ J

9. Ans. (B, C, D)

Sol.



$$1 \sin\phi = \frac{\sqrt{7}}{2}\sin r_1$$

since rays enters from air to glass $\Rightarrow r_2 \le \theta_c$

Thus no TIR from back surface

10. Ans. (B, D)

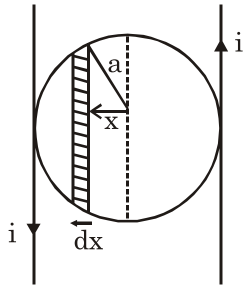
Sol. Since inductor is ideal $\Rightarrow$ EMF inductor = Potential difference inductor and EMF across resistor = 0

11. Ans. (A, D)

Sol. flux through the strip,

$$d\phi = \left[\frac{\mu_0}{2\pi}\frac{i}{(a-x)} + \frac{\mu_0}{2\pi}\frac{i}{(a+x)}\right]\left(2\sqrt{a^2-x^2}\right)dx$$

$$= \frac{\mu_0}{2\pi o} i \left[\frac{1}{a-x} + \frac{1}{a+x}\right] \times 2\sqrt{a^2-x^2}dx$$



$$d\phi = \frac{2\mu_0 ia}{\pi}.\frac{dx}{\sqrt{a^2-x^2}}$$

Total flux through the ring,

$$\phi = 2\left[\frac{2\mu_0 ia}{\pi}\int_0^a \frac{dx}{\sqrt{a^2-x^2}}\right]$$

$$= \frac{4\mu_0 ia}{\pi}\left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= = \frac{4\mu_0 ia}{\pi}\left[\frac{\pi}{2}\right]$$

$$\phi = 2\mu_0 ia$$

Mutual inductance, $M = \frac{Q_2}{i_\perp} = \frac{2\mu_0 ia}{i} = 2\mu_0 a$

Also, $M = \frac{\varepsilon_2}{di_\perp/dt} \Rightarrow \varepsilon_2 = \frac{Mdi_\perp}{dt}$

$$\varepsilon_2 = 2\mu_0 ai.e^{\left(\frac{2}{\mu_0}\right)t}.\frac{2}{\mu_0}$$

$$\varepsilon_2 = 4ai$$

Induced current in the ring, $i_R = \frac{\varepsilon_2}{R}$

(R resistance)

$$i_R = \frac{4ai}{2\pi a} = \frac{2i}{\pi}$$

Magnetic field at the centre,

$$B_{net} = \frac{\mu_0}{2\pi}.\frac{i}{a} + \frac{\mu_0}{2\pi}.\frac{i}{a} - \frac{\mu_0}{2}.\frac{(2i/\pi)}{a}$$

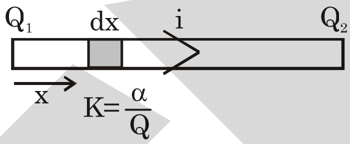
$$= \frac{\mu_0}{\pi}.\frac{i}{a} - \frac{\mu_0 i}{\pi a}$$

$$= 0$$

12. Ans. (D)

Sol. Current in the element,

$$i = -KA\frac{dQ}{dx}$$



$$i\int_0^\ell dx = -A \propto \int_{Q_1}^{Q_2}\frac{dQ}{Q}$$

$$i\ell = A \propto \ln\left(\frac{Q_1}{Q_2}\right) \qquad ...(i)$$

Also, $i\int_0^x dx = -A \propto \int_{Q_1}^{Q}\frac{dQ}{Q}$

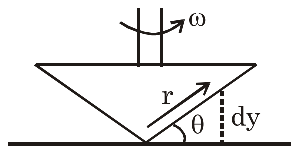
$$ix = A \propto \ln\left(\frac{Q_1}{Q}\right)$$

Using (i), $\left(\frac{Q_1}{Q_2}\right)^{x/\ell} = \frac{Q_1}{Q}$

$$Q = Q_1\left(\frac{Q_2}{Q_1}\right)^{x/\ell}$$

13. Ans. (A, C)

Sol. Velocity gradient



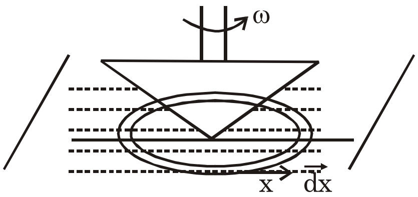
$$\frac{dv}{dy} = \frac{\omega r\cos\theta - 0}{r\sin\theta}$$

$$= \frac{\omega}{\tan\theta} = \frac{\omega}{\theta}$$

(as θ is small)

Considering elemental ring of radius x & thickness dx torque on this elemental ring,

$$d\tau = \left[(2\pi x dx)\frac{\omega}{\theta}\right]x\eta$$



$\Rightarrow$ Net torque, $\tau = \frac{2}{3}\pi R^3 \eta \frac{\omega}{\theta}$

14. Ans. (B, C, D)

Sol. $f = f_0\left(\frac{v - v_0}{v - v_s}\right)$

If source is receding away, then

$$f = f_0\left(\frac{v}{v + v_s}\right)$$

so f decreases as v_s increases

If observer receding away, then

$$f = f_0\left(\frac{v - v_0}{v}\right)$$

so, f decreases as v_0 increases but at high speeds f will increase and at very high speeds no frequency will be detected

15. Ans. (B)

Sol. $\rho g dy = -\frac{BdV}{V} = \frac{Bd\rho}{\rho}$

$Bd\rho = \rho^2 g\, dy$

$\rho = \rho + \frac{\rho^2 gh}{B}$

$= \rho\left(1 + \frac{\rho gh}{B}\right)$

$$\int \frac{dy}{\rho^2} = \int \frac{dy}{B}$$

$$-\left[\frac{1}{\rho} - \frac{1}{\rho_0}\right] = \frac{gh}{B}$$

$$\frac{1}{\rho_0} - \frac{1}{\rho} = \frac{gh}{B}$$

$$\frac{1}{\rho_0} - \frac{gh}{B} = \frac{1}{\rho}$$

$$\rho = \frac{B\rho_0}{B - \rho_0 gh} = \rho_0\left(1 - \frac{\rho_0 gh}{B}\right)^{-1}$$

$$= \rho_0\left(1 + \frac{\rho_0 gh}{B}\right)$$

16. Ans. (A)

Sol. $\frac{dp}{dy} = -\rho g = -g(\rho_0 + by)$

$dp = \rho_0 gh + b\frac{gh^2}{2}$

17. Ans. (B)

Sol. Least energetic photon corresponds to transition from n = 2 to n = 1,

so $\lambda = \frac{1500\,P^2}{P^2 - 1} = \frac{1500 \times 2^2}{2^2 - 1}$

$= 2000\,\text{Å} = 200\,\text{nm}$

18. Ans. (D)

Sol. $\Delta E_{21} = \frac{hc}{\lambda} = \frac{1240\,\text{eV} - \text{nm}}{200} = 6.2\,\text{eV}$

only in option (D), there is difference of 6.2 V in energy of first & second level.

i.e, $E_2 - E_1 = 6.2$ eV

PART-3 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B	D	C	D	A	C	A,B,C,D	A,B,C	A,B,C,D
	Q.	11	12	13	14	15	16	17	18		
	A.	B,C	A,D	A,D	A,B,C,D	D	B	B	B		

SOLUTION

SECTION-I

1. Ans. (D)

Sol. $[Ag^+]$ for pptⁿ of I^- = $\dfrac{k_{sp}(AgI)}{[I^-]}=\dfrac{10^{-17}}{0.1}=10^{-16}\,M$

$[Ag^+]$ for pptⁿ of Br^- = $\dfrac{k_{sp}(AgBr)}{[Br^-]}=\dfrac{10^{-13}}{0.1}=10^{-12}\,M$

⇒ $[I^-]$ where AgBr starts precipitating = $\dfrac{k_{sp}(AgI)}{(Ag^+)}=\dfrac{10^{-17}}{10^{-12}}=10^{-5}\,M$

⇒ % $[I^-]$ remaining in solution = $\dfrac{10^{-5}}{0.1}\times 100=0.01\%$

⇒ % $[I^-]$ precipitated =99.99%

2. Ans. (B)

Sol. ph = 3 ⇒ $[H^+] = 10^{-3}$ M

let the degree of dissociation be α

⇒ $\alpha=\dfrac{10^{-3}}{0.01}=0.1$

⇒ $i = 1 + (y - 1)\alpha = 1.1$

⇒ π(osmotic pressure) = (1.1) (0.01) RT = 0.011 RT

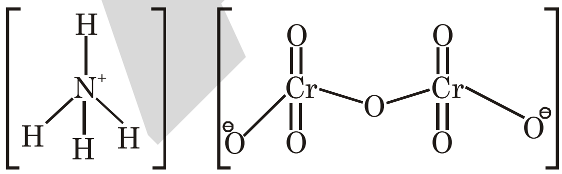
3. Ans.(D)

Sol. (A) $N_2H_4 + 2I_2 \longrightarrow 4HI + N_2$

(B) $N_2H_4 + 2O_2 \longrightarrow 2H_2O_2 + N_2$

(C) $N_2H_4 + 2CuSO_4 \longrightarrow Cu + N_2 + 2H_2SO_4$

4. Ans.(C)

Sol. (A) 2 

(B) $(NH_4)_2\,Cr_2O_7 \xrightarrow{\Delta} N_2 + \underset{\text{(green)}}{Cr_2O_3} + 4H_2O$ (this section is used for making artificial volcano)

(C) Metal nitrate $\xrightarrow{\Delta}$ metal oxide + NO_2 + O_2

(D) Metal nitrates are generally soluble in water

5. Ans. (D)

Sol. $CH_3-CH(CH_3)-CH_2-CH_3 \xrightarrow{Cl_2} CH_3-CH(CH_3)-CH_2-CH_2-Cl$

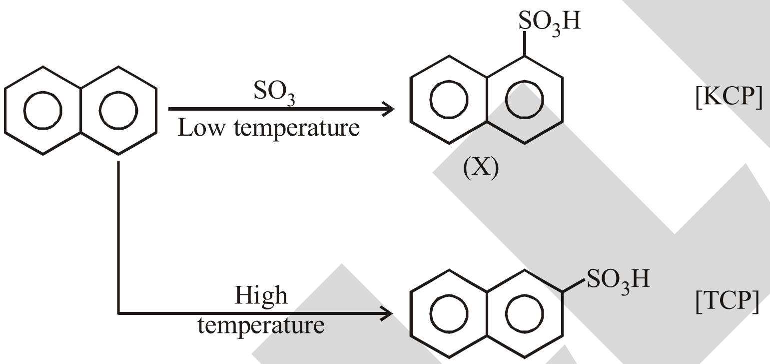
$CH_3-CH(CH_3)-CH(Cl)-CH_3$

$CH_3-C(CH_3)(Cl)-CH_2-CH_3$

$Cl-CH_2-CH(CH_3)-CH_2-CH_3$

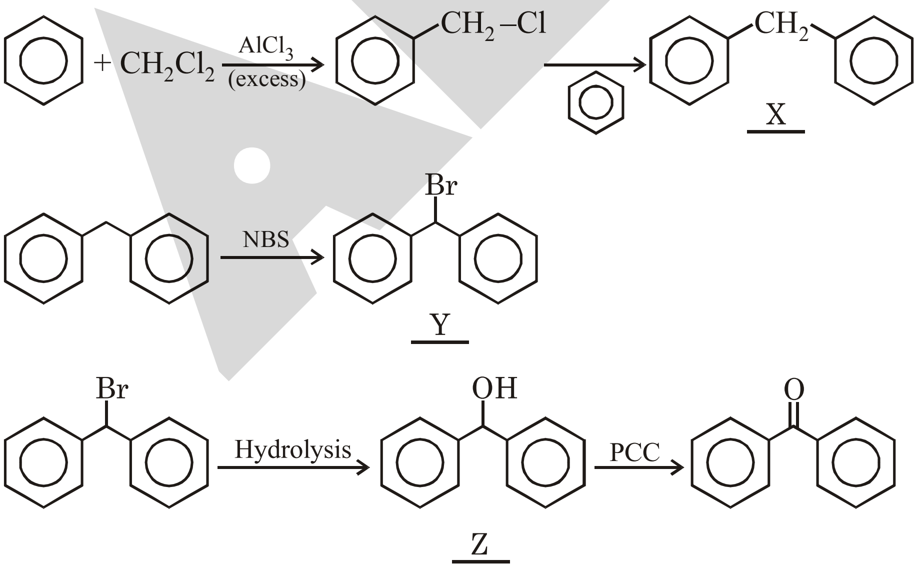
6. Ans. (A)

Sol.



7. Ans. (C)

Sol.



8. (A, B, C, D)
Refer theory

9. Ans. (A, B, C)

Sol. (A) Carbon atoms occupy all lattice points and also are present in alternate tetrahedral voids.

(B) Two atoms in the closest neighbourhood are the ones present at any lattice point and into the tetrahedral void.

$\Rightarrow$ $$d_{nearest} = 2r = \left(\frac{\sqrt{3}a}{8}\right) \times 2 = \frac{\sqrt{3}\,a}{4}$$

(C) Contribution from atoms at corner = $8 \times \frac{1}{8} = 1$

contribution from atoms at face centres = $6 \times \frac{1}{2} = 3$ contribution from atoms in half tetrahedral voids = $\frac{1}{2} \times 8 = 4$

$\Rightarrow$ Total atoms (effective) = 8

10. Ans.(A, B, C, D)

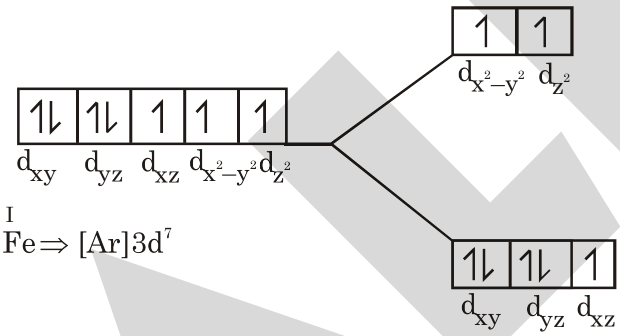
Sol. $NO_2^- + I^- \longrightarrow NO + I_2$

$\downarrow FeSO_4$

$[\overset{I}{Fe}(H_2O)_5\overset{\oplus}{N}O]SO_4$

Brown coloured complex

$[\overset{I}{Fe}(H_2O)_5\overset{\oplus}{N}O]SO_4$



$\overset{I}{Fe} \Rightarrow [Ar]3d^7$

$\mu = \sqrt{15}$ B.M., brown coloured complex due to charge transfer, hybridisation $\Rightarrow sp^3d^2$

11. Ans.(B, C)

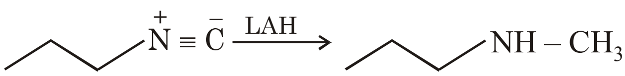
Sol. (A) Na^+ & $K^{\oplus}$ ions can form alums.

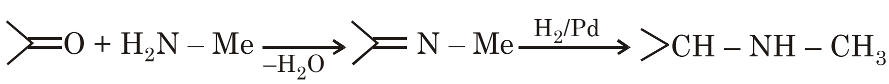
(B) $K_2SO_4.Al_2(SO_4)_3.24H_2O \xrightarrow{BaCl_2} \underset{\text{white ppt.}}{BaSO_4} \xrightarrow{\text{dil. } HNO_3}$ Not Soluble

(C) Alums are used as a mordants in dyeing industry.

(D) All types of alums are not coloured.

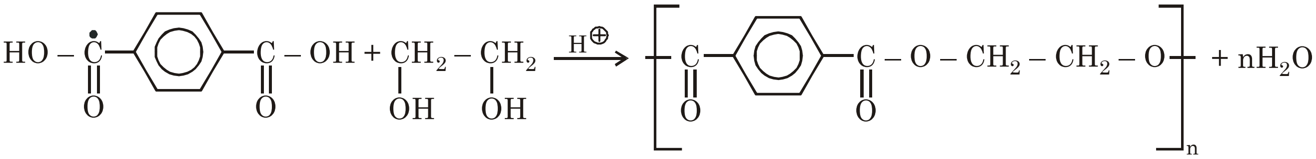
12. Ans.(A, D)

Sol. (A) 

(D) 

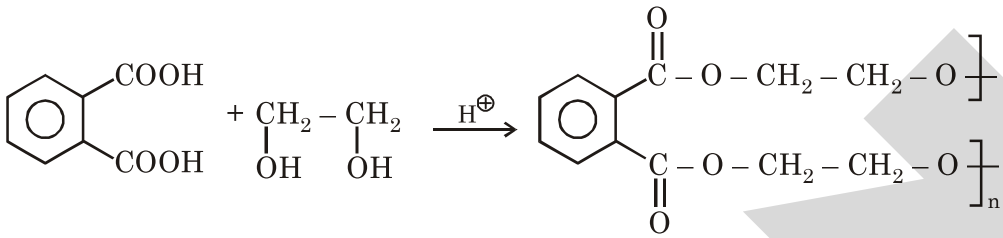
13. Ans. (A, D)

Sol. (A)



N Mole

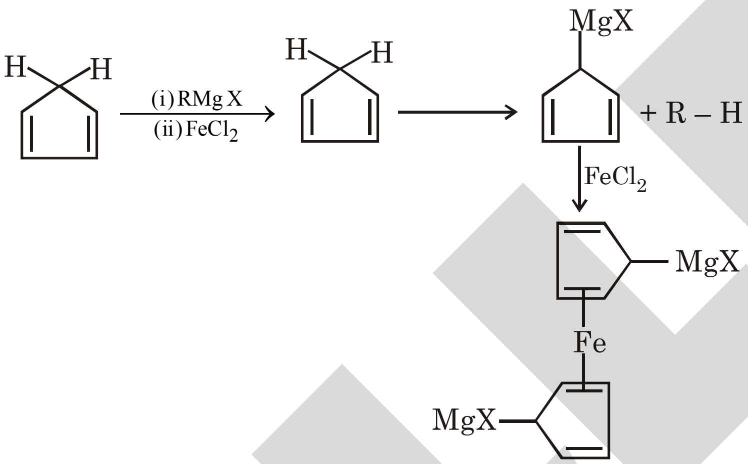
(D)



Pthallic Acid Glyptal

14. Ans. (A, B, C, D)

Sol.



15. Ans. (D)

Sol. m mol of compound = $\frac{300\,mg}{150\,g/mol} = 2$

m mole of H_2O produced = $\frac{324\ mg}{18\ g/mol} = 18$

$\Rightarrow$ m mole of H atom = (18 × 2)

$\Rightarrow$ m mole of H atom in one m mole of compound = $\frac{18\times 2}{2} = 18$

m mole of C in 2 m mole compound = m mole of H_2CO_3

$= \frac{m\,mole\,of\,NaOH}{2} = \frac{0.3\times 80}{2} = 12\,m\,mole$

$\Rightarrow$ m mole of C in 1 m mole compound = 6

m mole of $N_2 = \left(\frac{95}{760}\right)\left(\frac{394}{0.0821}\right)\times\frac{1}{300} \simeq 2$

$\Rightarrow$ m mole of N present in 1 m mole of compound = 2

$\Rightarrow$ $C_6H_{18}O_x.N_2 \equiv$ mol.wt = (6 × 12 + 18 × 1 + 16 × x + 14 × 2) = 150

$\Rightarrow$ x = 2

$\Rightarrow$ formula = $C_6H_{18}N_2O_2$

16. Ans. (B)

Sol. $n_{CO_2} = n_{H_2CO_3} = \left(\frac{12}{1000}\right)$

$n_{N_2} = \frac{2}{1000}$

$\Rightarrow \quad n_{CO_2} + n_{N_2} = 0.014$

17. Ans.(B)

Sol. (A) No change observed in test tubes.

(B) $Sn^{+2} \xrightarrow[\text{Basic medium}]{H_2S} \underset{\text{Brown ppt}}{SnS\downarrow}$

$Sn^{+4} \xrightarrow[\text{Basic medium}]{H_2S} \underset{\text{Yellow ppt}}{SnS_2\downarrow}$

(C) $Sn^{+2} \xrightarrow{\text{dil. NaOH}} \underset{\text{white ppt}}{Sn(OH)_2\downarrow}$

$Sn^{+2} \xrightarrow{\text{dil. NaOH}} \underset{\text{white ppt}}{Sn(OH)_4\downarrow}$

(D) $Sn^{+2} \xrightarrow{NaOH} \underset{\text{white ppt}}{Sn(OH)_2\downarrow} \xrightarrow[\text{excess}]{NaOH} \underset{\text{soluble}}{[Sn(OH)_4]^{-2}}$

$Sn^{+2} \xrightarrow{NaOH} \underset{\text{white ppt}}{Sn(OH)_4\downarrow} \xrightarrow[\text{excess}]{NaOH} \underset{\text{soluble}}{[Sn(OH)_6]^{-2}}$

18. Ans.(B)

Sol. when tin(II) & tin (IV) compound dissolve in excess NaOH then $[Sn(OH)_4]^{-2}$ & $[Sn(OH)_6]^{-2}$ ions are formed respectively.